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Sultan, I., Reda, A., & Forbes, F. (2012). A surrogate model for evaluation of maximum normalized dynamic load factor in moving load model for pipeline spanning due to slug flow. *31st International Conference on Ocean, Offshore and Arctic Engineering*

Which has been published in final form at:

<http://doi.org/10.1115/OMAE2012-83746>

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A SURROGATE MODEL FOR EVALUATION OF MAXIMUM NORMALIZED DYNAMIC LOAD FACTOR IN MOVING LOAD MODEL FOR PIPELINE SPANNING DUE TO SLUG FLOW

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ABSTRACT

Understanding the problem of slug-flow-induced fatigue damage is of particular importance to the reliable operation of pipelines. Slug flow, across unsupported pipeline spans, produces dynamic vibrations in the pipeline resulting in cyclical fatigue stresses. The dynamic effects will cause the pipeline to fail at a point of stress concentration if proper design procedure is not followed.

The response of a pipeline span, under the passage of slug flow, can be represented by dynamic load factors that are functions of the speed ratio and damping characteristics of the span. The aspects of these functional relationships are investigated, in this paper, by conducting multiple simulations at different speed ratios and damping factors. The data obtained from the steady state Fourier expansion will, consequently, be used to produce a surrogate model with a level of accuracy that adequately qualifies it for use in determining dynamic loading of pipelines.

The closed-form surrogate model can be used to eliminate the need to employ costly mathematical procedures or finite element packages for the analysis. The model will also provide a solid ground for optimization studies and help designers gain an insight into how various model parameters impact the system response. This paper will demonstrate the aspects of a proposed surrogate model and endeavor to obtain parameter domains within which the model's reliability is ensured. A

numerical example will be demonstrated to prove the concepts presented in the paper and confirm the validity of the proposed model.

INTRODUCTION

Slugging is one of the operational hazards of subsea pipelines associated with the transportation of multiphase fluids. Growth in the use of pipelines that operate in a multiphase flow regime, has allowed further complications in subsea pipeline's design. Slug flow across unsupported pipeline spans produces vertical vibration, resulting in dynamic stresses; ultimately the pipeline will fail at the girth weld location, especially in a corrosive environment. Therefore, it is crucial that pipeline engineers identify potential fatigue damage arising from slug flow to ensure pipeline integrity will not be compromised.

Reda et al (2011) demonstrated that the response of a simply supported beam under a concentrated moving force displays a response that is qualitatively similar to the single degree of freedom system. This will only occur if the structure has a light damping and the vibration modes are well separated. Reda et al (2012) verified further that the relationship between the maximum normalized dynamic load factor (DLF) and speed parameters, for a given damping ratio, does not change with the span length or the pipeline cross section. Reda et al (2012) also concluded that the moving force model is a valid model in many situations and yields excellent results when compared to the moving mass model.

The intention of this paper is to develop a simplified surrogate model to determine the maximum normalized DLF of both vertical displacement and bending moment. This can then be used to determine the bending stresses required to perform the fatigue life estimates of pipelines. The proposed model can be employed at the early stages of the design when numbers of different options are investigated. Moreover, the model can be used in the detailed design stages providing that the slug problem is treated as a moving point force. A surrogate model can also be used to quickly perform sensitivity analyses to investigate the effects of uncertain input parameters such as slug speed, slug length and pipe wall thickness. Additionally, the model can be used for initial validation of finite element calculations.

Sultan (2007) suggests that surrogate models are used where the exact mathematical form of a relationship, underlying a certain process, is not readily obtainable. In this case, empirical methods are usually employed together with suitable regression techniques to work out a substitution functional expression relating the various parameters controlling the process. Surrogate models are used for a broad range of applications from simulating the effects of some engine parameters on its performance (Bates et al, 2000) to describing machining errors in turning processes (Gilsinn et al, 2002). The process of obtaining a fitted model is essentially an optimization exercise, where the difference between the estimated system output and its measured value is to be minimized. The optimization procedure adopted in this paper is based on the approach of Simultaneous Perturbation Stochastic Approximation (SPSA). This approach is efficient and suited for such an intricate optimization application as abundantly explained in literature, e.g. the excellent paper by (Spall, 1998) and the interesting application tackled by (Kothandaraman and Rotea, 2005).

To obtain data sets of input parameters needed for the analysis, some researchers employ the method of Monte Carlo simulation (Jeang et al, 2002) or the concept of response surface (Carley et al, 2004 and Robinson et al, 2004).

The surrogate model presented in this paper covers a wide range of applications such as railways, road bridges and many other structural situations that involve a moving load problem. This is because the proposed model is expressed as a function of non-dimensional parameters.

METHODOLOGY

A surrogate model can be defined as a non-linear inverse problem where a continuous function of a set of design parameters is to be defined. Figure 1 illustrates the steps adopted here for the construction of a surrogate model for the DLF for both displacement and bending moment in a beam with a moving concentrated force travelling across its span. The steps adopted for model design are as follows.

Step 1 – Data Collection: the steady state time-based Fourier expansion, presented below, for the beam responses (both

vertical deflection and bending moment) is obtained and employed in a suitable optimization technique to find the maximum values of these response. These maximum values will be registered for the speed parameter range of 0.3 to 1.5 and for the damping factor range of 0.02 to 0.2. These ranges have been selected because they encompass many practical applications encountered by design engineers. Expanded ranges will be attempted in further publications.

Once the data points are generated, it is advisable to produce a set of graphs to inform the process of proposing a mathematical formulation for the surrogate model.

Step 2 – Model Construction: guided by the graphs obtained in the first step, propose a surrogate model.

Step 3 – Optimization: is concerned with finding the coefficients of the proposed equation such that the surrogate model reflects the behavior of the original system as closely as possible. This step is executed here via a suitable stochastic procedure.

Figure 1: Steps Adopted to Construct the Design Equations of DLF for Displacement and Bending Moment

Step 4 – Validation: The purpose of this step is to establish the predictive capabilities of the surrogate model, once it is available. If the outcome from the validation process is satisfactory then the surrogate model can be used. If not, then Steps 2 and 3 will be repeated via iterative processes until the surrogate model is acceptable.

STEP 1 – SIMULATION

Figure 2 illustrates a simply supported beam, with length L , subject to concentrated force P travelling at a constant

velocity, v . When the force is located at distance x_p from the left hand side shoulder of the pipeline span, a functional relationship can be written in the following form to express the beam deflection, y , at any point, C , which falls at a distance, x from the left hand side shoulder of the span:

Figure 2: Simply Supported Beam Subjected to a Contracted Load

$$y(\alpha, \beta, \varphi, \mu) = y_0 \sum_{j=0}^{\infty} [f_j(\alpha, \beta, \varphi, \mu)] \quad (1)$$

The above equation is due to Fryba (1999) and its parameters are defined as follows;

j : is the mode number.

$y_0 \approx \frac{2PL^3}{\pi^4 EI}$: is the static deflection calculated at the center of the beam.

Where: E is Young's modulus and I is the second moment of area of the beam cross-section.

The non-dimensional function $f_j(\alpha, \beta, \varphi, \mu)$ in equation (1) can be expressed as follows:

$$f_j(\alpha, \beta, \varphi, \mu) = \left[g_1(\alpha, \beta, j) \cdot \sin(j \cdot \alpha \cdot \varphi + \gamma_1(\alpha, \beta, j)) - g_2(\alpha, \beta, j) \cdot e^{-\beta \varphi} \cdot \sin \left[\sqrt{(j^2 - \beta^2)} \cdot \varphi + \gamma_2(\alpha, \beta, j) \right] \right] \cdot \sin(j \cdot \pi \cdot \mu) \quad (2)$$

Where:

β is the non-dimensional damping factor given by:

$$\beta = \frac{\omega_b}{\omega_1} \quad (3)$$

ω_1 is the fundamental natural frequency of the simply supported beam.

$$\omega_1 = \frac{\pi^2}{L^2} \sqrt{\frac{EI}{\rho_A}} \quad (4)$$

Where: ρ_A the mass per unit length of the beam

ω_b is the damping frequency.

α is a non-dimensional speed parameter which is calculated by:

$$\alpha = \frac{\omega}{\omega_1} \quad (5)$$

ω is the circular frequency of slug defined by:

$$\omega = \frac{\pi v}{L} \quad (6)$$

φ signifies the load position, from the LHS side, and it can be calculated in accordance to the following:

$$\varphi = \left(\frac{\pi}{\alpha} \right) \left(\frac{x_p}{L} \right) \quad (7)$$

μ defines the x -location on the beam for the point at which the deflection calculations are being performed. In this context μ can be defined as follows:

$$\mu = \frac{x}{L} \quad (8)$$

The functions included in the f_j expression are given as follows:

$$\gamma_1(\alpha, \beta, j) = -\tan^{-1} \left[\frac{2\alpha\beta}{j(j^2 - \alpha^2)} \right] \quad (9)$$

$$\gamma_2(\alpha, \beta, j) = -\tan^{-1} \left[\frac{2\beta\sqrt{j^4 - \beta^2}}{j^2(j^2 - \alpha^2) - 2\beta^2} \right] \quad (10)$$

$$\gamma_3(\alpha, \beta, j) = -\tan^{-1} \left[\frac{\sqrt{j^2 - \beta^2}}{\beta^2} \right] \quad (11)$$

$$g_1(\alpha, \beta, j) = \frac{1}{j\sqrt{j^2(j^2 - \alpha^2)^2 + 4\alpha^2\beta^2}} \quad (12)$$

$$g_2(\alpha, \beta, j) = \frac{\alpha}{j[j^2(j^2 - \alpha^2)^2 + 4\alpha^2\beta^2]} \sqrt{\frac{j^2(j^2 - \alpha^2) - 2\beta^2}{j^4 - \beta^2}} + 4\beta^2 \quad (13)$$

The equation for bending moment is expressed as follows:

$$M(\alpha, \beta, \varphi, \mu) = -\frac{EI}{L^2} \frac{d^2 y(\alpha, \beta, \varphi, \mu)}{d\mu^2} \quad (14)$$

Combining equations (1) and (14), the following expression may be written for the bending moment::

$$M(\alpha, \beta, \varphi, \mu) = \frac{\pi^2 EI y_0}{L^2} \sum_{j=0}^{\infty} [j^2 f_j(\alpha, \beta, \varphi, \mu)] \quad (15)$$

Equation (15) determines the bending moment, M , at the location defined by parameter, μ and load location defined by φ .

In this paper, $y(\alpha, \beta, \varphi, \mu)$ and $M(\alpha, \beta, \varphi, \mu)$ are replaced by their normalized dimensionless counterparts, $y_n(\alpha, \beta, \varphi, \mu)$ and $M_n(\alpha, \beta, \varphi, \mu)$, as follows;

$$y_n(\alpha, \beta, \varphi, \mu) = y(\alpha, \beta, \varphi, \mu)/y_0 \quad (16)$$

and

$$\mathbf{M}_n(\alpha, \beta, \varphi, \mu) = \frac{L^2}{\pi^2 E I y_0} \mathbf{M}(\alpha, \beta, \varphi, \mu) \quad (17)$$

At a given pair of α and β , an optimization technique is adopted to find the positions of φ_y and μ_y at which the maximum beam deflection occurs:

$$\begin{aligned} &\text{Maximize } y_n(\alpha, \beta, \varphi, \mu)^2 \\ &\text{Subject to } \begin{cases} \frac{\pi}{\alpha} \geq \varphi \geq 0 \\ 1 \geq \mu \geq 0 \end{cases} \end{aligned} \quad (18)$$

Consequently, the resulting values, φ_y and μ_y , can be used to calculate the maximum normalized beam deflection, $y_{n-max}(\alpha, \beta)$ which corresponds to the given values of α and β as follows;

$$y_{n-max}(\alpha, \beta) = y_n(\alpha, \beta, \varphi_y, \mu_y) \quad (19)$$

Similarly, at a given pair of α and β , an optimization technique is adopted to find the positions of φ_M and μ_M at which the maximum beam bending moment occurs:

$$\begin{aligned} &\text{Maximize } \mathbf{M}_n(\alpha, \beta, \varphi, \mu)^2 \\ &\text{Subject to } \begin{cases} \frac{\pi}{\alpha} \geq \varphi \geq 0 \\ 1 \geq \mu \geq 0 \end{cases} \end{aligned} \quad (20)$$

Consequently, the resulting values, φ_M and μ_M , can be used to calculate the maximum normalized bending moment, $M_{n-max}(\alpha, \beta)$ which corresponds to the given values of α and β as follows;

$$\mathbf{M}_{n-max}(\alpha, \beta) = \mathbf{M}_n(\alpha, \beta, \varphi_M, \mu_M) \quad (21)$$

It should be pointed out here that $y_{n-max}(\alpha, \beta)$ and $M_{n-max}(\alpha, \beta)$ are the Dynamic Load Factors (DLF) for the deflection and bending moment respectively.

Figure 3 below describes the relationship between μ_y and α as drawn at a selected range of β . As can be seen from the figureFigure 3, the maximum DLF of displacement or $y_{n-max}(\alpha, \beta)$ does not necessarily occur at the mid-point of the beam. It can be seen that the location of the maximum displacement alternates around the mid-point of the beam between μ_y equals to 0.53 and 0.44, approximately $\pm 5\%$ around the mid-point. The location of maximum displacement depends on α and β .

Figure 3: Relationship between Normalized μ_y and α @ Selected Range of β

Figure 4 describes the relationship between the normalized φ_y and α drawn at a selected range of β . It is worth noting that φ_y has been normalized by multiplying it by α and dividing it by π (i.e. $\varphi_{y-normalized} = \varphi_y \frac{\alpha}{\pi}$.)

Figure 4: Relationship between Normalized φ_y and α @ Selected Range of β

Figure 4 is beneficial to engineers who are interested in knowing the location of the concentrated moving force in relation to the LHS of the beam, when the maximum deflection occurs along the beam. For example, for a speed ratio of 0.7 and a damping ratio of 0.2, from Figure 3, it can be concluded that maximum displacement will take place at 53% of the span length. From Figure 4, it can be concluded that the maximum deflection occurs when the point force is located at 80% of the span length. This is can be schematically explained by Figure 5 below.

Figure 5: A Schematic Diagram Showing the Location of the Force at the Maximum Bending Moment.

Figure 7: Relationship between Normalized ϕ_M and α @ Selected Range of β

In a similar fashion to Figure 3 and Figure 4, Figure 6 can be used to determine the location of the maximum bending moment, whereas Figure 7 is used to determine the location of the point force, in relation to the LHS of the beam when the maximum bending moment occurs.

Figure 8 and Figure 9 illustrate how α impacts $y_{n-max}(\alpha, \beta)$ and $M_{n-max}(\alpha, \beta)$, respectively, at a selected range of β .

Figure 6: Relationship between Normalized μ_M and α @ Selected Range of β

Figure 6 describes the relationship between μ_M and α drawn at a selected range of β . As can be seen from the figure, the maximum bending DLF or $M_{n-max}(\alpha, \beta)$ does not always occur at the mid-point of the beam. It can be seen that the location of the maximum bending moment alternates around the mid-point of the beam between μ_y equals to 0.53 and 0.44, approximately $\pm 21\%$ around the mid-point. The location of maximum displacement depends on α and β . Also, the location of the maximum bending moment is much more sensitive to the speed parameter than is the displacement.

Figure 7 describes the relationship between the normalized ϕ_M and α drawn at a selected range of β . Similar to ϕ_y , ϕ_M has been normalized by multiplying it by α and dividing it by π (i.e. $\phi_{M-normalized} = \phi_M \frac{\alpha}{\pi}$).

Figure 8: Relationship between y_{n-max} and α @ Selected Range of β

Figure 9: Relationship between M_{n-max} and α @ Selected Range of β

STEP 2 – CONSTRUCT SURROGATE MODEL

A set of numerical experiments were conducted to propose equations (22) and (23) below, to describe how α and β impact the values of $y_{n-max}(\alpha, \beta)$ and $M_{n-max}(\alpha, \beta)$ as follows;

$$y_s(\alpha, \beta) = \frac{a_0[(\alpha^{a_1} + a_2\alpha^{a_3}\beta^{a_4})e^{-\beta\alpha a_5 + a_6}]e^{-\alpha b_0}}{\sqrt{\alpha^{b_1} + b_2\alpha + b_3\alpha^{b_4}\beta^{b_5} + b_6}} \quad (22)$$

$$M_s(\alpha, \beta) = \frac{c_0[(\alpha^{c_1} + c_2\alpha^{c_3}\beta^{c_4})e^{-\beta\alpha c_5 + c_6}e^{-\alpha d_0}]}{\sqrt{\alpha^{d_1} + d_2\alpha + d_3\alpha^{d_4}\beta^{d_5} + d_6}} \quad (23)$$

where: $y_s(\alpha, \beta)$ and $M_s(\alpha, \beta)$, respectively, signify the estimated values of $y_{n-max}(\alpha, \beta)$ and $M_{n-max}(\alpha, \beta)$ which correspond to given values of α and β . In (22) and (23), the coefficients $a_0, a_1, \dots, a_6, b_0, b_1, \dots, b_6, c_0, c_1, \dots, c_6$ and $d_0, d_1, \dots, d_6$ are to be found using a suitable optimization technique. These coefficients are dimensionless and represent small numerical contributions to the relationships shown in Figure 7 and Figure 8.

STEP 3 – OPTIMIZATION

The procedure in (18) and (20) was used to generate 336 various data points for each of $y_{n-max}(\alpha, \beta)$ and $M_{n-max}(\alpha, \beta)$ at different pairs of α and β . This was performed in order to find the coefficients in equations (22) and (23). The procedures adopted to calculate the optimal values of these coefficients are as follows;

$$\text{Minimize } \sum w_{yk}(y_{n-max}(\alpha_k, \beta_k) - y_s(\alpha_k, \beta_k))^2 \quad (24)$$

$$\text{Minimize } \sum w_{Mk}(M_{n-max}(\alpha_k, \beta_k) - M_s(\alpha_k, \beta_k))^2 \quad (25)$$

where: k is the number of data points, w_{yk} and w_{Mk} are weights given to specific data points for the deflection and bending moment respectively.

Mathematically, the optimization procedures in (24) and (25) involve the minimization of a cost function, $F(\theta)$, as follows;

$$\begin{aligned} &\text{Minimise: } F(\theta) \\ &\text{Subject to } \theta_{\min} \leq \theta \leq \theta_{\max} \end{aligned} \quad (26)$$

where: $\theta_{\min}$ and $\theta_{\max}$ are respectively the minimum and maximum constraints imposed on the design parameters.

As revealed in the introduction, the approach of SPSA is adopted here for the optimization procedure. In accordance with this approach, the updated values of the design parameter, θ_q^{i+1} , which occupies the position number q in the vector of coefficients is calculated at the end of iteration step number i as follows;

$$\theta_q^{i+1} = \theta_q^i - r_i \frac{[F(\theta^i + C_i \Delta^i) - F(\theta^i - C_i \Delta^i)]}{2C_i \Delta^i} \quad (27)$$

where Δ is a vector whose entries are randomly assigned the values of either -1 or +1 as generated, at every iteration, by a binary Bernoulli distribution. The parameters, r_i and C_i , in equation (27) are the sequence gains, which are calculated at iteration number i as follows;

$$r_i = R/(B + i)^{0.602} \quad (28)$$

$$C_i = C/(i)^{0.101} \quad (29)$$

(Spall, 1998) points out the guidelines that can be followed to select numerical values for the constants, R and C , in equations (28) and (29). The value of B is calculated as $K/10$, where K is the maximum allowable number of iterations as set at the start of the procedure. The limits imposed on the values of the design parameter θ_q (i.e. θ_{q-max} and θ_{q-min}) are incorporated in the procedure, as suggested by (Kothandaraman and Rotea, 2005), as follows;

$$\theta_q^{i+1} = \begin{cases} \theta_{q-max} & \text{if } \theta_q^{i+1} > \theta_{q-max} \\ \theta_{q-min} & \text{if } \theta_q^{i+1} < \theta_{q-min} \end{cases} \quad (30)$$

The outcome of the optimization procedure is then used for model validation to ensure the suitability for the proposed model for the intended purpose.

STEP 4 – SURROGATE MODEL VALIDATION

To validate the surrogate models derived in equations (22) and (23), 100 data points, for each of the vertical deflection and bending moment, were randomly generated, in the desired ranges of α and β , using a Monte Carlo simulator. At each data point, the exact values of $y_{n-max}(\alpha, \beta)$ and $M_{n-max}(\alpha, \beta)$ were obtained as pointed out in Step 1. The corresponding values $y_s(\alpha, \beta)$ and $M_s(\alpha, \beta)$ were obtained from equations (22) and (23) to establish a basis for comparison. Figure 10 and 11 below, respectively, compare $y_{n-max}(\alpha, \beta)$ to $y_s(\alpha, \beta)$ and $M_{n-max}(\alpha, \beta)$ to $M_s(\alpha, \beta)$.

Figure 10: Relationship between Normalized y_{max} and α @ Selected Range of β Calculated using Exact and Surrogate Models

Figure 11: Relationship between Normalized $M_{\max}$ and α @ Selected Range of β Calculated using Exact and Surrogate Models

The percentage errors at data point number i , $(ery)_i$ and $(erM)_i$, in the fitted values of $y_s(\alpha, \beta)$ and $M_s(\alpha, \beta)$ are calculated as follows:

$$(ery)_i = \left| \frac{y_s(\alpha_i, \beta_i) - y_{n-\max}(\alpha_i, \beta_i)}{y_{n-\max}(\alpha_i, \beta_i)} \right| \times 100 \quad (31)$$

$$(erM)_i = \left| \frac{M_s(\alpha_i, \beta_i) - M_{n-\max}(\alpha_i, \beta_i)}{M_{n-\max}(\alpha_i, \beta_i)} \right| \times 100 \quad (32)$$

Figure 12 and Figure 13 depict the errors incurred by using the proposed surrogate models for both the vertical deflection and bending moment respectively.

Figure 12: A Spectrum of the Deflection Absolute Error for Randomly Generated Data Points

Figure 13: A Spectrum of the Bending Moment Absolute Error for Randomly Generated Data Points

Histograms are plotted for the percentage error as shown in Figure 14 and Figure 15 for both the vertical deflection and bending moment respectively. The average error in the vertical deflection is 1.67% and its corresponding standard deviation is 1.12%. The corresponding value for the bending moment error is 2.5% and 1.57% respectively. The small error values confirm the validity of the fitted model and its suitability to replace the complicated procedure, involved in solving the original model equations, by simplified mathematical equations that are solved by straightforward substitutions.

Figure 14: A Histogram of the Deflection Error for Randomly Generated Data Points

Figure 15: A Histogram of the Bending Moment Error for Randomly Generated Data Points

RESULTS

The optimization procedure yielded the following values for the design equations coefficients for evaluation of maximum normalized DLF in moving load model for pipeline spans due to slug flow.

Table 1: Displacement Equation (22)

a₀	2.198	b₀	0.923
a₁	0.876	b₁	3.913
a₂	3.167	b₂	4.104
a₃	0.890	b₃	2.720
a₄	-0.072	b₄	1.563
a₅	0.497	b₅	1.937
a₆	0.125	b₆	3.881

Table 2: Bending Moment Equation (23)

c₀	3.634	d₀	0.172
c₁	-0.172	d₁	-2.125
c₂	-0.683	d₂	10.071
c₃	10.382	d₃	-0.239
c₄	6.833	d₄	3.086
c₅	2.841	d₅	6.998
c₆	0.854	d₆	4.847

It should be highlighted that the proposed design equations are only valid for the parameters shown in Table 3.

Table 3: Parameters Included in Simulation

Parameter	Range
α -Speed Parameter	0.3-1.5
β - Damping Ratio	0.02-0.2
D/t	Applicable only for thin-walled pipe
Span Length	< 100 m

WORKED EXAMPLES

In this section the maximum normalized DLF for the displacement and the bending moment will be calculated in accordance to equations (22) and (23) respectively. The aim of this worked example is to calculate the stress range required to perform the fatigue assessment at the girth weld. Girth weld is typically the weakest link in a pipeline.

Table 4: Input Data

Parameter	Range
Pipe Outside Diameter (D)	323.9 mm
Wall Thickness (t)	12.7 mm
Material Young's Modulus (E)	205 GPa
Steel Density (ρ)	7850 kg/m ³
Pipe Unit Weight	97.47 kg/m
Total Damping Ratio β	0.2
Speed Velocity (V)	18 m/sec
Slug Weight (W)	143.5 kg
Span Length (L)	30 m

The solution can be summarized in the following steps:

1. Calculate the critical speed:

$$u_{cr} = \frac{\pi}{L} \left(\frac{EI}{\rho A} \right)^{0.5} = 58.93 \text{ m/sec}$$

2. Calculate the speed parameter:

$$\alpha = \frac{V}{u_{cr}} = 0.3$$

3. Substitute α , β and the design equations coefficients in equations (22) and (23) to determine the maximum normalized DLF of displacement and bending moment respectively. The resulting $y_s(\alpha, \beta)$ is 1.209 and $M_s(\alpha, \beta)$ is 1.473.

4. Calculate the static displacement and the static bending moment using the following equations:

$$y_0 \approx \frac{2WL^3}{\pi^4 EI} = 0.026 \text{ m}$$

$$\frac{\pi^2 EI y_0}{L^2} = 8.555 \text{ kNm}$$

5. Calculate the estimated maximum vertical displacement and estimated maximum bending moment experienced by the pipeline spanning due to the slug, using the following:

$$y_{max} = y_s(\alpha, \beta) \times y_0 = 0.031 \text{ m}$$

$$M_{max} = M_s(\alpha, \beta) \times \frac{\pi^2 EI y_0}{L^2} = 12.6 \text{ kNm}$$

6. Obtain the location of the maximum displacement and bending moment from Figure 3 and Figure 4 respectively at damping ratio 0.2.

The exact values calculated from the original mathematical procedure for y_{max} and M_{max} are 0.036 m and 12.0 kNm respectively.

It should be pointed out that the estimated maximum vertical displacement and bending moment represent the upper-bound envelope for both of them. This is based on the recent work carried out by (Reda and Forbes, 4), where they demonstrated that, the concentrated force model over-predicts the response for a distributed load.

Once the displacement and the actual maximum bending moment are determined, the bending stress can be calculated based on the calculated bending moment; consequently, the fatigue assessment is based on the number of cycles of slug frequency.

CONCLUSIONS

A surrogate model has been proposed in this paper to calculate the maximum vertical deflection and bending moment occurring in a vibrating pipeline. The proposed model was found to be accurate enough in the specified parameter space.

The design equations presented in this paper will ensure much faster and straightforward calculations for the fatigue damage associated with slug flow across unsupported span. Moreover, these equations can be used at the early stages of the design process when numbers of different options are being considered. The proposed design equations can also be used in detailed design stages providing the slug problem can be treated as a moving force problem. These design equations can be used to quickly perform sensitivity analyses to investigate the effects of uncertain input parameters such as slug speed, slug length and pipe wall thickness. Additionally, the design equations can be used to validate finite element calculations.

Reda and Forbes (4) showed that for a distributed load, the concentrated force model over-predicts the response and is almost identical for load distribution lengths less than 10% of the span length. This suggests that the maximum normalized dynamic load factor, calculated by the proposed design equations, presents the upper-bound of maximum dynamic load factor.

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