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Optimum design of limaçon gas expanders based on thermodynamic performance¹

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ABSTRACT

Positive displacement expanders are acquiring popularity due to the current push to harvest energy from low-grade heat resources which have been previously overlooked. The limaçon technology does offer a simple and reliable design with a considerable potential for small-size ($\leq 4\text{kW}$) power plants. This paper presents a thermodynamic model for the limaçon design and goes on to utilise this model in an optimisation procedure adopted to calculate the expanders geometric parameters for specific power and operating constraints. The numerical method employed to solve the thermodynamic model is presented for the benefit of the reader. Two design case studies, for expanders with and without an inlet control valve, are offered at the end of the paper to prove the validity of the presented concepts and their suitability for the analysis.

1. INTRODUCTION

Like their turbo-machinery counterparts, positive displacement expanders are used to extract mechanical energy from pressurised gaseous fluids. However, unlike turbo-machines, Lemort et al [1] point out that positive displacement expanders are suitable for low speed, low flow rate and high pressure ratio applications. Moreover, Smith

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and Stosic [2] argue that these expanders can handle two-phase flows better than conventional turbines.

Current interest in positive displacement gas expanders is fuelled by the need to capture sources of energy which may have been previously overlooked in two main areas; namely, refrigeration and power generation. Refrigeration plants based on CO₂ exhibit low COP values; and in order to increase their economic viability the pressure energy lost to the throttling process may be recovered by gas expanders as demonstrated by a number of authors, e.g. Nagata et al [3] and Smith and Stosic [2]. The other application which requires the use of positive displacement expanders are the power production plants which are based on the Organic Rankine Cycle (ORC). These plants utilise organic fluids to extract energy from low-grade heat sources as detailed by Angelino et al [4]. Organic fluids are most suitable for this application because of their low boiling temperatures as has been highlighted by Mago et al [5] who report a very interesting observation that these plants perform best when the working fluid is at saturated conditions before it enters the expander. This underlines the need for positive displacement expanders which are naturally capable of handling two-phase flows expected to take place in these power production plants. Doty and Shevgoor [6], in the context of introducing the dual-source ORC, present an insightful discussion on organic fluids and their limitations as working media for Organic Rankine Cycles.

Most small ORC-based power plants utilise scroll-type positive displacement expanders which apply isentropic expansion on the working fluid without the need for an inlet valve. However, the modelling and experimental work, by Lemort et al [1, 7], underlines leakage as a problem which these expanders suffer even when special arrangements have been implemented to reduce this leakage mechanically. On the other hand, the results presented by Wang et al [8] suggest that their method to suppress leakage by applying an external pressure on the mating surfaces does help to improve the expander volumetric performance. Due to the isentropic expansion that takes place inside the chamber, good efficiency figures have been reported for scroll expanders. These figures range from 63% reported by Saitoh et al [9] to 83% reported by Mathias et al [10]. Another expander design which does not require an

inlet valve is the vane-type expander described by Yang et al [11] who demonstrated a successful effort to improve its isentropic efficiency from 9% to 23%.

Expander designs which require inlet valve control to improve their efficiency have been reported by a number of authors. For example Baek et al [12] report an effort to utilise a piston-cylinder arrangement for expanding CO₂ in refrigeration plants. The discussion presented by Baek et al [12] on the workings and requirements of inlet control valves is indeed of particular importance to future developments in the area of gas expanders. Other gas expander innovative designs featured in the literature are the rolling-piston design by Li et al [13]; and the free-piston design Zhang et al [14].

This paper is intended to reflect on the geometric and thermodynamic workings of the limaçon positive displacement gas expander. Sultan [15] lists a number of available limaçon designs dated as far back as the late 1800's which have not been utilised by industry due to lack of understanding of their geometric and manufacturing aspects. Recent effort to develop a better understanding of these machines has been published by Sultan [15, 16 and 17] and Sultan and Schaller [18]. As such, limaçon machines are now on the path to attract industrial attention and acquire popularity for power production and utilisation. The paper will present a thermodynamic model for the limaçon expander. A model which will then solved numerically and utilised in an optimisation procedure to calculate the optimum geometric parameters of the machine to meet some specified power and operating constraints. The optimisation method of Simultaneous Perturbation Stochastic Approximation (SPSA) which has been presented by Spall [19] will be used for the optimisation procedure. This method, despite its simplicity, is suited for intricate process optimisation problems such as the one described in this paper. The geometric aspects of the limaçon machine will be described in the next section.

2. THE LIMAÇON WORKING CHAMBER

The main geometric features of a limaçon machine are detailed in Figure 1. As shown in the figure, the rotor (whose chord length is $2L$) is made to rotate and slide about

the limaçon pole (point o), which is the origin of a stationary Cartesian frame, XY. On the other hand, another Cartesian frame, X_rY_r, is rigidly attached to the rotor at its centre and moves with it. As the rotor moves, its centre point, m, stays kinematically attached to the housing base circle whose radius is given as r . The limaçon is in fact the curve traced by the apices, p_l and p_r , on the rotor centreline, which is also known as the limaçon chord. By virtue of the geometric properties of the limaçon curve, the two apices are always touching the housing wall, and slide on it in the directions defined by the tangents at the points of contact. This ensures good sealing action and smooth contact conditions. To prevent the rotor-housing interference, Sultan (2006) proposes applying a clearance, C_r , to the rotor flank; and/or employing a slightly larger rotor base circle radius, r_r , than the one used for the housing base circle. Consequently, the following expression is given for the size of working chamber volume, V_c , at any rotor angle θ ;

$$V_c = H_r L^2 \left[\pi \left(b^2 - \frac{r_r^2}{L^2} \right) + \pi \frac{C_r}{L} \left(1 - \frac{1}{2} \frac{C_r}{L} \right) + 4 \frac{r_r}{L} \left(1 - \frac{C_r}{L} \right) - 4b \cos \theta \right] \quad (1)$$

where H_r is the axial depth of the rotor measured perpendicular to the page and b (where $0 < b \leq 0.25$) is used to replace the ratio of r/L ; i.e. the limaçon aspect ratio.

The derivative, $dV_c/d\theta$, impacts the performance of limaçon machines. This derivative can be expressed as follows;

$$dV_c/d\theta = 4bL^2 H_r \sin \theta \quad (2)$$

The next section introduces the model presented in this paper to calculate the instantaneous port area as seen from the working chamber being studied.

3. PORT AREA CALCULATIONS

The limaçon machine has two ports; one for fluid admission and one for discharge. The concepts presented in this section will apply to both ports without discrimination; and as such, no specific reference will be made to either port in the section.

Figure 2 depicts a port on a limaçon machine with two position vectors, $\mathbf{R}_l$ and $\mathbf{R}_t$, used to define the radial positions of the port leading and trailing edges respectively. The leading edge is the one which the rotor's leading apex, p_l , meets first during one cycle of operation. The port edge vectors can be expressed as follows;

$$\begin{aligned}\mathbf{R}_l &= L(2b \sin \theta_l + 1)\hat{\mathbf{R}}_l \quad \text{and} \\ \mathbf{R}_t &= L(2b \sin \theta_t + 1)\hat{\mathbf{R}}_t\end{aligned}\tag{3}$$

where the angular locations of the leading and trailing edges are, respectively, defined by the angles, θ_l and θ_t . Usually, θ_l is assigned first and then a port angular width, $\Delta\theta_p$, is determined. The angular position of the trailing edge is then calculated from $\theta_t = \theta_l + \Delta\theta_p$. If θ_t is found to be greater than 2π , then 2π has to be subtracted from the calculated value of θ_t . The unit vectors, $\hat{\mathbf{R}}_l$ and $\hat{\mathbf{R}}_t$, used in equation (3) are given as follows;

$$\begin{aligned}\hat{\mathbf{R}}_l &= [\cos \theta_l \quad \sin \theta_l \quad 0]^T \quad \text{and} \\ \hat{\mathbf{R}}_t &= [\cos \theta_t \quad \sin \theta_t \quad 0]^T\end{aligned}\tag{4}$$

The port length, L_p , is normally given within the constraints imposed by the rotor depth. The width, W , is calculated from the two edge vectors as follows;

$$W = |\mathbf{R}_t - \mathbf{R}_l|\tag{5}$$

Taking the port ends to be semicircular, the full area, A_f , of the port may be expressed as follows;

$$A_f = L_p W - W^2 \left(1 - \frac{\pi}{4}\right)\tag{6}$$

The rotor-port interaction is studied here in relation to the location of the leading apex of the rotor with respect to the port edges. For this purpose we define the position of the rotor's leading apex using the vector $\mathbf{P}_l$ as follows;

$$\mathbf{P}_l = L(2b \sin \theta + 1) \hat{\mathbf{X}}_r \quad (7)$$

where the unit vector $\hat{\mathbf{X}}_r$ is given as follows;

$$\hat{\mathbf{X}}_r = [\cos \theta \quad \sin \theta \quad 0]^T \quad (8)$$

The relative locations of the rotor leading apex with respect to the port edges can now be defined using the signs of two scalar quantities, s_l and s_t , which are given as follows;

$$\begin{aligned} s_l &= (\hat{\mathbf{R}}_l \times \hat{\mathbf{X}}_r) \cdot (\hat{\mathbf{X}}_r \times \hat{\mathbf{Y}}_r) \quad \text{and} \\ s_t &= (\hat{\mathbf{R}}_t \times \hat{\mathbf{X}}_r) \cdot (\hat{\mathbf{X}}_r \times \hat{\mathbf{Y}}_r) \end{aligned} \quad (9)$$

where the unit vector $\hat{\mathbf{Y}}_r$ is given as follows;

$$\hat{\mathbf{Y}}_r = [-\sin \theta \quad \cos \theta \quad 0]^T \quad (10)$$

Sultan and Schaller [18] suggest the following algorithm to calculate the instantaneous value of the port area, A_p ,

$$\text{for } s_l \geq 0 \begin{cases} \text{if } s_t \geq 0 & A_p = A_f \\ \text{if } s_t < 0 & A_p = (W_l / W) A_f \end{cases} \quad (11)$$

and

$$\text{for } s_l < 0 \begin{cases} \text{if } s_t \leq 0 & A_p = 0 \\ \text{if } s_t > 0 & A_p = (W_t / W) A_f \end{cases} \quad (12)$$

where $W_l = |\mathbf{P}_l - \mathbf{R}_l|$, $W_t = |\mathbf{P}_t - \mathbf{R}_t|$ and the position vector $\mathbf{P}_t$ is given as follows;

$$\mathbf{P}_t = L(2b \sin \theta - 1) \hat{\mathbf{X}}_r \quad (13)$$

The velocity of flow blowing through the downstream side of a port is presented in the next section.

4. VELOCITY OF FLOW THROUGH PORTS

Flow through the ports of a limaçon machine exhibits a change of direction as does the flow through a globe valve. As such, these ports are approximated here as globe valves rather than orifice plates. An approximation which is supported by the work published by McNeil [20]. Essentially, flow through ports undergoes an energy conversion process manifested by the velocity resulting on the downstream side of the port. The ideal value of this velocity may be given for compressible flow as $\sqrt{2\Delta h}$ where Δh is an isentropic enthalpy drop taken in the direction of flow. However, this energy conversion process is accompanied by losses due to friction and cross sectional variations. The well known loss coefficient is employed here to approximate the effects of these losses. Massoud [21] points out that the loss coefficient, K_p , can be expressed as

$$K_p = N_p f_p \quad (14)$$

where N_p is a specific number given for each type of flow obstruction and f_p is a friction factor expressed for turbulent flow by

$$f_p = 0.184/R_e^{0.2} \quad (15)$$

where R_e is the Reynolds number. Utilising the well known definition of the Reynolds number, the velocity, U_p , of flow blowing through the downstream side of a port may now be expressed as follows;

$$U_p = \begin{cases} \left(\left(\frac{\rho_{pd} D_{pd}}{\mu_{pd}} \right)^{0.1} \sqrt{\frac{2(h_{pu} - h_{pd})}{0.184 N_p}} \right)^{10/9} & \text{if } \sqrt{2(h_{pu} - h_{pd})} < U_{ps} \\ \left(\frac{\rho_{pd} D_{pd}}{\mu_{pd}} \right)^{0.1} \frac{U_{ps}^{1.1}}{\sqrt{0.184 N_p}} & \text{otherwise} \end{cases} \quad (16)$$

where the letter p which appears in the subscripts may be replaced by i for the inlet port and o for the outlet port. Also, u and d signify upstream and downstream respectively. The variable h_{pu} is the specific stagnation enthalpy on the upstream side of a port and h_{pd} is the specific enthalpy on its downstream side. The downstream density, viscosity and hydraulic diameter are given, respectively, as ρ_{pd} , μ_{pd} and D_{pd} . In the above equation, h_{pd} and ρ_{pd} are calculated by considering an isentropic expansion process from upstream to downstream; and U_{ps} is the speed of sound on the downstream side of the port. Equation (16) is employed for both the single phase and two phase flow situations. For a two phase flow, however, μ_{pd} may be calculated in accordance with the McAdams formula,

$$\frac{1}{\mu_{pd}} = \frac{x}{\mu_{vap}} + \frac{1-x}{\mu_{liq}} \quad (17)$$

where x is the dryness fraction and the subscripts vap and liq refer to vapour and liquid respectively. Also for a two phase flow the speed of sound may be calculated based on the assumption that the fluid is in equilibrium, over pressure and temperature, as detailed in the excellent paper by Lund and Flätten [22].

As suggested by equation (16), the use of loss coefficient, whilst being a reasonable approximation, makes it possible to calculate flow velocities in a non-iterative fashion. Also, the use of enthalpy difference in the velocity equation instead of pressure difference eliminates the need to calculate the expansion factor. This is useful for situations in which a two phase flow may take place during the expansion process. In the following section, the flow velocity on the downstream side of a port, U_p , will be employed in the continuity and energy equations to obtain a differential thermodynamic model for the system. In that context, U_p , will be referred to as U_i and U_o to associate the velocity with either the inlet or outlet ports, respectively.

5. THERMODYNAMIC MODEL

The rate, dm_c/dt , at which the mass inside a chamber varies in relation to density and available volume, V_c . This flow rate can be obtained from the following equation,

$$dm_c/dt = \omega \left[V_c (d\rho_c/d\theta) - 4\rho_c b L^2 H_r \sin \theta \right] \quad (18)$$

where the subscript c signifies chamber fluid; and m_c and ρ_c are the mass and density of this fluid, respectively, at any crank angle θ . In the model presented here, dm_c/dt denotes derivative with respect to time and ω is a constant rotor velocity given in rad/sec . The change occurring to the chamber mass is created by the flow through the inlet and outlet ports. As such, the continuity equation can be written for the chamber which falls below the rotor in the following form;

$$\frac{d\rho_c^B}{d\theta} = \left(\frac{1}{\omega V_c^B} \right) \left\{ A_i^B \rho_{id}^B U_i^B - A_o^B \rho_{od}^B U_o^B - \dot{m}_s^B \mp \dot{m}_{ap} + 4\rho_c^B b \omega L^2 H_r \sin \theta \right\} \quad (19)$$

where the superscript, B , denotes, "below rotor". The inlet and outlet port areas, A_i and A_o respectively, are calculated as described above. The density and velocity on the downstream side of the inlet port are given, respectively, as ρ_{id} and U_i . The corresponding quantities on the downstream side of the outlet port are given as ρ_{od} and U_o , respectively. Both U_i and U_o are calculated as described by equation (16). In the above equation, $\dot{m}_s$ and $\dot{m}_{ap}$, respectively, signify leakage past the side and apex seals. The $\pm$ sign is used in front of $\dot{m}_{ap}$ to highlight the fact that the direction of leakage, from one chamber to another, is determined by the instantaneous value of pressure in each chamber. The corresponding continuity equation for the working chamber which falls above the rotor is written as follows;

$$\frac{d\rho_c^A}{d\theta} = \left(\frac{1}{\omega V_c^A} \right) \left\{ A_i^A \rho_{id}^A U_i^A - A_o^A \rho_{od}^A U_o^A - \dot{m}_s^A \mp \dot{m}_{ap} + 4\rho_c^A b \omega L^2 H_r \sin \theta \right\} \quad (20)$$

Taking the energy transfer to and from a working chamber fluid as an adiabatic process will result in the following equation;

$$dH_i/dt - dH_o/dt = d(m_c e_c)/dt + P_c dV_c/dt \quad (21)$$

where H_i and H_o are the total enthalpies moving in and out of a working chamber, respectively; and e_c is the specific internal energy available in the working chamber. The term $d(m_c e_c)/dt$ can be substituted by $d(m_c h_c - P_c V_c)/dt$ and $m_c dh_c/dt$ is in turn given by $m_c T_c ds_c/dt + V_c dP_c/dt$; where T_c and s_c are, respectively, the temperature and entropy inside the working chamber; and h_c is the specific enthalpy of the chamber fluid. For the working chamber below the rotor, it should now be possible to manipulate (21) into the following form;

$$\begin{aligned} \frac{ds_c^B}{d\theta} = \frac{1}{\omega \rho_c^B T_c^B V_c^B} & \left(A_i^B U_i^B \rho_{id}^B (h_{iu}^B - h_c^B) - A_o^B U_o^B \rho_{od}^B (h_{ou}^B - h_c^B) \right. \\ & \left. - \dot{m}_s^B (h_{su}^B - h_c^B) \pm \dot{m}_{ap} (h_{apu}^B - h_c^B) \right) \end{aligned} \quad (22)$$

where h_{iu} and h_{ou} refer to the specific enthalpies on the upstream side of the inlet and outlet ports respectively. Also, h_{su} and h_{tu} signify the specific enthalpies on the upstream sides of the side and apex seals respectively. The corresponding equation for the working chamber which falls above the rotor is given as follows;

$$\begin{aligned} \frac{ds_c^A}{d\theta} = \frac{1}{\omega \rho_c^A T_c^A V_c^A} & \left(A_i^A U_i^A \rho_{id}^A (h_{iu}^A - h_c^A) - A_o^A U_o^A \rho_{od}^A (h_{ou}^A - h_c^A) \right. \\ & \left. - \dot{m}_s^A (h_{su}^A - h_c^A) \mp \dot{m}_{ap} (h_{apu}^A - h_c^A) \right) \end{aligned} \quad (23)$$

Now the four simultaneous differential equations, (19), (20), (22) and (23), can be solved numerically to find instantaneous values for s_c^A , s_c^B , ρ_c^A and ρ_c^B at corresponding values for the crank angle, θ . At every step, the corresponding working chamber pressure can be obtained using the function, $\text{Pressure}(s_c, \rho_c)$,

offered by REFPROP 8.0, Lemmon et al [23]. This function produces accurate results for both single and two phase flows. The working chambers temperature and specific enthalpy can, respectively, be obtained by $\text{Temperature}(P_c, s_c)$ and $\text{Enthalpy}(P_c, s_c)$. Whilst all the given conditions in the inlet manifold are taken to be constant, only the pressure, P_o , is assumed to be maintained constant in the outlet manifold. The entropy in this manifold, s_o , can be obtained by mixing the flows blowing out of the two working chambers. As such, the temperature is calculated by $\text{Temperature}(P_o, s_o)$ and the density is obtained from $\text{Density}(P_o, s_o)$. The same approach has been applied to the conditions inside the rotor side cavity which houses the driving mechanism.

At every iteration, the instantaneous value of shaft torque, τ_{sh} , which results from the fluid pressure has been given by Sultan [15] as follows;

$$\tau_{sh} = 4bL^2H_r \left(P_c^B - P_c^A \right) \sin \theta \quad (24)$$

The iterative approach proceeds by calculating the thermodynamic model at small intervals in the range $\theta_{li} \leq \theta \leq \theta_{li} + \pi$, where the θ_{li} defines the angular position of the inlet port leading edge. The values, $P_c^B(\theta_{li})$ and $\rho_c^B(\theta_{li})$, assumed for the pressure and density below the rotor, respectively, at the start of the cycle are compared to the corresponding values, $P_c^A(\theta_{li} + \pi)$ and $\rho_c^A(\theta_{li} + \pi)$, calculated above the rotor. Also, the values, $P_c^A(\theta_{li})$ and $\rho_c^A(\theta_{li})$, assumed for the pressure and density above the rotor, respectively, at the start of the cycle are compared to the corresponding values, $P_c^B(\theta_{li} + \pi)$ and $\rho_c^B(\theta_{li} + \pi)$, below the rotor using the following dimensionless error expression;

$$\sigma = \left[\left(\frac{P_c^B(\theta_{li}) - P_c^A(\theta_{li} + \pi)}{\bar{P}_{c1}} \right)^2 + \left(\frac{\rho_c^B(\theta_{li}) - \rho_c^A(\theta_{li} + \pi)}{\bar{\rho}_{c1}} \right)^2 + \left(\frac{P_c^A(\theta_{li}) - P_c^B(\theta_{li} + \pi)}{\bar{P}_{c2}} \right)^2 + \left(\frac{\rho_c^A(\theta_{li}) - \rho_c^B(\theta_{li} + \pi)}{\bar{\rho}_{c2}} \right)^2 \right]^{1/2} \quad (25)$$

where $\bar{P}_{c1}$, $\bar{P}_{c2}$, $\bar{\rho}_{c1}$ and $\bar{\rho}_{c2}$ are, respectively, pressure and density values used for error calculations. These values are obtained as follows;

$$\bar{P}_{c1} = \frac{P_c^B(\theta_{li}) + P_c^A(\theta_{li} + \pi)}{2} \quad (26)$$

$$\bar{\rho}_{c1} = \frac{\rho_c^B(\theta_{li}) + \rho_c^A(\theta_{li} + \pi)}{2} \quad (27)$$

$$\bar{P}_{c2} = \frac{P_c^A(\theta_{li}) + P_c^B(\theta_{li} + \pi)}{2} \quad (28)$$

$$\bar{\rho}_{c2} = \frac{\rho_c^A(\theta_{li}) + \rho_c^B(\theta_{li} + \pi)}{2} \quad (29)$$

If the outcome of equation (25) is larger than a small predefined value, $P_c^B(\theta_{li})$, $\rho_c^B(\theta_{li})$, $P_c^A(\theta_{li})$ and $\rho_c^A(\theta_{li})$ are set, respectively, equal to $P_c^A(\theta_{li} + \pi)$, $\rho_c^A(\theta_{li} + \pi)$, $P_c^B(\theta_{li} + \pi)$ and $\rho_c^B(\theta_{li} + \pi)$ to repeat the procedure again over the π range of θ . This is iterated until the calculated error, σ , falls within an acceptable range to reflect the cyclical nature of the thermodynamic process. Once convergence has been achieved, the total energy, E_{cyc} , per one cycle is calculated using numerical integration as follows;

$$E_{cyc} = -2\delta\theta \left(\sum_{n=0}^N (\tau_{sh})_n - \frac{(\tau_{sh})_N + (\tau_{sh})_0}{2} \right) \quad (30)$$

where $\delta\theta$ is the size of the angular interval, n is a counter for successive points on the curve being integrated and N is the total number of intervals on the curve. In a like manner, the total mass flow through the machine in one cycle, M_{cyc} , is calculated by the following expression;

$$M_{cyc} = 2 \frac{\delta\theta}{\omega} \left\{ \sum_{n=0}^N \left(A_i^B \rho_{id}^B U_i^B \right)_n - \left(A_i^A \rho_{id}^A U_i^A \right)_n - \frac{\left(A_i^B \rho_{id}^B U_i^B \right)_N + \left(A_i^B \rho_{id}^B U_i^B \right)_0}{2} - \frac{\left(A_i^A \rho_{id}^A U_i^A \right)_N + \left(A_i^A \rho_{id}^A U_i^A \right)_0}{2} \right\} \quad (31)$$

It is worthy of noting here that the expressions in (30) and (31) feature a multiplication by 2 in order to account for the fact that a limaçon machine does produce two cycles for every full shaft rotation.

6. MODEL SOLUTION

The simplest method to solve the four simultaneous differential equations, (19), (20), (22) and (23), is to use an Euler forward substitution. However, this straightforward technique may not be sufficiently stable for thermodynamic problems where some system parameters approach steady state at much faster rates than others. To ensure some level of stability for this method, the crank cyclical displacement, π , has to be divided into a huge number of sections before applying the iterative technique. However, with the implementation of the an external library, such as REFPROP 8.0, in the solution, the computational cost of having a large number of divisions on the θ -axis may prove prohibitive. As such a simple predictor-corrector procedure, based on a modified form of the Euler implicit solution, is adopted here for the analysis. To detail this numerical approach, the following definitions are presented;

$$\left. \begin{aligned} \frac{d\rho_c^B}{d\theta} &= f_{rB} \\ \frac{ds_c^B}{d\theta} &= f_{sB} \\ \frac{d\rho_c^A}{d\theta} &= f_{rA} \\ \frac{ds_c^A}{d\theta} &= f_{sA} \end{aligned} \right\} \quad (32)$$

At step number $n+1$, on the θ -axis, it is required to calculate, $s_c^{B^{n+1}}$, $\rho_c^{B^{n+1}}$, $s_c^{A^{n+1}}$ and $\rho_c^{A^{n+1}}$, which are the instantaneous values of s_c^B , ρ_c^B , s_c^A and ρ_c^A respectively. It is assumed here that the corresponding values have been already calculated, at step

n , and they are given, respectively, as $s_c^{B^n}$, $\rho_c^{B^n}$, $s_c^{A^n}$ and $\rho_c^{A^n}$. The values of their corresponding derivatives are also known and referred to as f_{sB}^n , f_{rB}^n , f_{sA}^n and f_{rA}^n .

To this end, the modified Euler substitution can be used for the solution as follows;

$$\rho_c^{B^{n+1}} = \rho_c^{B^n} + (\delta\theta/2)(f_{rB}^{n+1} - f_{rB}^n) \quad (33)$$

$$s_c^{B^{n+1}} = s_c^{B^n} + (\delta\theta/2)(f_{sB}^{n+1} - f_{sB}^n) \quad (34)$$

$$\rho_c^{A^{n+1}} = \rho_c^{A^n} + (\delta\theta/2)(f_{rA}^{n+1} - f_{rA}^n) \quad (35)$$

$$s_c^{A^{n+1}} = s_c^{A^n} + (\delta\theta/2)(f_{sA}^{n+1} - f_{sA}^n) \quad (36)$$

It can readily be noticed that the pair $(\rho_c^{B^{n+1}}, f_{rB}^{n+1})$ occurs simultaneously; and so do $(s_c^{B^{n+1}}, f_{sB}^{n+1})$, $(\rho_c^{A^{n+1}}, f_{rA}^{n+1})$ and $(s_c^{A^{n+1}}, f_{sA}^{n+1})$. Therefore, a nested Newton-Raphson iterative formulation is adopted here to obtain the system solution at step number $n+1$. As such, at nested iteration number j , a solution for the roots of equations (33) to (36) can be manipulated to the following forms

$$\rho_{c_j}^{B^{n+1}} = \rho_c^{B^n} + \frac{\delta\theta(\rho_{c_{j-1}}^{B^{n+1}} - \rho_c^{B^n})f_{rB}^n}{(\rho_{c_{j-1}}^{B^{n+1}} - \rho_c^{B^n}) - (\delta\theta/2)(f_{rB_{j-1}}^{n+1} - f_{rB}^n)} \quad (37)$$

$$s_{c_j}^{B^{n+1}} = s_c^{B^n} + \frac{\delta\theta(s_{c_{j-1}}^{B^{n+1}} - s_c^{B^n})f_{sB}^n}{(s_{c_{j-1}}^{B^{n+1}} - s_c^{B^n}) - (\delta\theta/2)(f_{sB_{j-1}}^{n+1} - f_{sB}^n)} \quad (38)$$

$$\rho_{c_j}^{A^{n+1}} = \rho_c^{A^n} + \frac{\delta\theta(\rho_{c_{j-1}}^{A^{n+1}} - \rho_c^{A^n})f_{rA}^n}{(\rho_{c_{j-1}}^{A^{n+1}} - \rho_c^{A^n}) - (\delta\theta/2)(f_{rA_{j-1}}^{n+1} - f_{rA}^n)} \quad (39)$$

$$s_{c_j}^{A^{n+1}} = s_c^{A^n} + \frac{\delta\theta(s_{c_{j-1}}^{A^{n+1}} - s_c^{A^n})f_{sA}^n}{(s_{c_{j-1}}^{A^{n+1}} - s_c^{A^n}) - (\delta\theta/2)(f_{sA_{j-1}}^{n+1} - f_{sA}^n)} \quad (40)$$

where $j=1,2,\dots$ and $\rho_{c_0}^{B^{n+1}}$, $s_{c_0}^{B^{n+1}}$, $\rho_{c_0}^{A^{n+1}}$ and $s_{c_0}^{A^{n+1}}$ are predicted, respectively, as;

$$\rho_{c_0}^{B^{n+1}} = \rho_c^{B^n} + \delta\theta f_{rB}^n \quad (41)$$

$$s_{c_0}^{B^{n+1}} = s_c^{B^n} + \delta\theta f_{sB}^n \quad (42)$$

$$\rho_{c_0}^{A^{n+1}} = \rho_c^{A^n} + \delta\theta f_{rA}^n \quad (43)$$

$$s_{c_0}^{A^{n+1}} = s_c^{A^n} + \delta\theta f_{sA}^n \quad (44)$$

The stopping criterion used for the nested iterations is given as follows;

$$\sqrt{\varepsilon_{rB}^2 + \varepsilon_{sB}^2 + \varepsilon_{rA}^2 + \varepsilon_{sA}^2} \leq \varepsilon \quad (45)$$

where ε is a small acceptable error value. The deviations, ε_{rB} , ε_{sB} , ε_{rA} and ε_{sA} are given at the end of each nested iteration number j by the following expressions;

$$\varepsilon_{rB} = \left(\rho_{c_j}^{B^{n+1}} - \rho_c^{B^{n+1}} \right) - (\delta\theta / 2) (f_{rB_j}^{n+1} + f_{rB}^n) \quad (46)$$

$$\varepsilon_{sB} = \left(s_{c_j}^{B^{n+1}} - s_c^{B^{n+1}} \right) - (\delta\theta / 2) (f_{sB_j}^{n+1} + f_{sB}^n) \quad (47)$$

$$\varepsilon_{rA} = \left(\rho_{c_j}^{A^{n+1}} - \rho_c^{A^{n+1}} \right) - (\delta\theta / 2) (f_{rA_j}^{n+1} + f_{rA}^n) \quad (48)$$

$$\varepsilon_{sA} = \left(s_{c_j}^{A^{n+1}} - s_c^{A^{n+1}} \right) - (\delta\theta / 2) (f_{sA_j}^{n+1} + f_{sA}^n) \quad (49)$$

The optimisation model which is used to design the limaçon gas expander is given in the next section.

7. OPTIMISED DESIGN FOR LIMAÇON GAS EXPANDERS

In this paper, the optimum expander is the one running at a given speed and capable of utilising available fluid at preset conditions to produce a required amount of indicated power, P_{ind} , with the highest possible isentropic efficiency, η_i , and a filling factor, ψ , as close to unity as possible. These performance metrics (P_{ind} , η_i and ψ) are function of the vector, Θ , of the expander geometric parameters. In the context of the simulation presented in this paper, P_{ind} is calculated by multiplying the energy produced in every revolution by the number of revolutions per second (i.e. $P_{ind} = E_{cyc} \omega / 2\pi$); and η_i is calculated as follows;

$$\eta_i = \Delta 00 E_{cyc} / M_{cyc} h_{io} \quad (50)$$

where Δh_{io} is the specific isentropic enthalpy drop from the inlet to the outlet manifolds. On the other hand, the filling factor, ψ , is given as follows;

$$\psi = \frac{M_{cyc}}{2\rho_i (V_c(\theta_{co}) - V_c(0))} \quad (51)$$

where ρ_i is the fluid density in the inlet manifold and θ_{co} is the angle at which the inlet control valve cuts off the flow. If an inlet control valve is not used, θ_{co} is set equal to π .

As such an objective function, $F(\Theta)$, is proposed as follows;

$$F(\Theta) = \sqrt{w_1 (1 - \psi)^2 + w_2 \left(1 - \frac{\eta_i}{100}\right)^2 + w_3 \left(1 - \frac{P_{req}}{P_{ind}}\right)^2} \quad (52)$$

where P_{req} is the power required from the expander. The positive weighting factors, w_1 , w_2 and w_3 are assigned subjectively to indicate the importance of each performance metric to the design at hand.

An eight-element vector, Θ , of geometric design parameters is given as follows;

$$\Theta = [L \quad b \quad L_i \quad L_o \quad \theta_{li} \quad \Delta \theta_i \quad \theta_{lo} \quad \Delta \theta_o]^T \quad (53)$$

where L and b are as defined previously. The inlet and outlet port lengths are given, respectively, as L_i and L_o . Other the port-related aspects which are used as model design parameters are θ_{li} and $\Delta \theta_i$, which are the angular position of the leading edge and the port angular width respectively. The corresponding design parameters for the outlet port are θ_{lo} and $\Delta \theta_o$. The optimisation problem can be posed by,

$$\begin{aligned} &\text{Minimize: } F(\Theta) \\ &\text{Subject to: } \Theta_{\min} \leq \Theta \leq \Theta_{\max} \end{aligned} \quad (54)$$

where $\Theta_{\min}$ and $\Theta_{\max}$ are respectively the minimum and maximum constraints imposed on the design parameters.

As revealed in the introduction, the approach of Simultaneous Perturbation Stochastic Approximation is adopted here for the optimisation procedure. In accordance with this approach, the updated values of the design parameter, Θ_q^{k+1} , which occupies the position number q in the design vector is calculated at the end of iteration step number k as follows;

$$\Theta_q^{k+1} = \Theta_q^k - a_k \left[F(\Theta^k + C_k \Delta^k) - F(\Theta^k - C_k \Delta^k) \right] / (2C_k \Delta_q^k) \quad (55)$$

where Δ is an eight-element vector whose entries are randomly assigned the values of either +1 or -1 as generated, at every iteration, by a binary Bernoulli distribution. The parameters a_k and C_k in equation (55) are the sequence gains which are calculated at iteration number k as follows;

$$\begin{aligned} a_k &= A/(B+k)^{0.602} \\ C_k &= C/(k)^{0.101} \end{aligned} \quad (56)$$

Spall [19] points out the guidelines which should be followed to select numerical values for the constants, A and C , in equation (56). For the work presented here, which features a low-noise application, C has been set equal to 0.0005 and A is set equal to 0.125. The value of B is calculated as $K/10$, where K is the maximum allowable number of iterations set at the start of the procedure. The limits imposed on the values of the design parameter Θ_q (i.e. $\Theta_q|_{\max}$ and $\Theta_q|_{\min}$) are incorporated in the procedure, as suggested by Kothandaraman and Rotea [24], as follows;

$$\Theta_q^{k+1} = \begin{cases} \Theta_q|_{\max} & \text{if } \Theta_q^{k+1} > \Theta_q|_{\max} \\ \Theta_q|_{\min} & \text{if } \Theta_q^{k+1} < \Theta_q|_{\min} \end{cases} \quad (57)$$

A flowchart of the computational procedure presented in this paper is depicted in Figure 3, and case studies are given in the next section to demonstrate the implementation of this procedure.

8. CASE STUDY 1. PORTED EXPANDER

For this study an expander is to be designed to utilise a supply of R-245fa available, as a working fluid, at an absolute pressure of 10 bar and a temperature of 150°C . The expander, which is not equipped with an inlet control valve, will be running at a speed of 1000 rpm and will expand the working fluid to 1 bar maintained in the outlet manifold. The power desired from the system is 3.0 kW. However, to account for possible mechanical losses, 3.5 kW should be produced by the expander. Taking into account that fact that the optimisation procedure will only yield a compromise amongst the various competing parameters of the system, a value of 4 kW will be fed to the simulation as input data. For mechanical reasons, the rotor axial depth, H_r , was assigned as βL , where $\beta = 1.3$. The limits imposed on the geometric parameters are given as follows;

$$40\text{ mm} \leq L \leq 200\text{ mm}, \quad 0.02 \leq b \leq 0.2, \quad 0.2\beta L \leq L_i \leq 0.7\beta L, \quad 0.2\beta L \leq L_o \leq 0.7\beta L, \\ -25^{\circ} \leq \theta_{li} \leq 25^{\circ}, \quad 5^{\circ} \leq \Delta\theta_i \leq 40^{\circ}, \quad 140^{\circ} \leq \theta_{lo} \leq 205^{\circ} \text{ and } 5^{\circ} \leq \Delta\theta_o \leq 50^{\circ}.$$

The initial values of the parameters are given as follows;

$$L = 40\text{ mm}, \quad b = 0.02, \quad L_i = 0.02\beta L, \quad L_o = 0.7\beta L, \quad \theta_{li} = -25^{\circ}, \quad \Delta\theta_i = 40^{\circ}, \quad \theta_{lo} = 205^{\circ} \\ \text{and } \Delta\theta_o = 50^{\circ}.$$

The optimisation procedure produced the following values for design parameters;

$$L = 46.45\text{ mm}, \quad b = 0.172, \quad L_i = 13.35\text{ mm}, \quad L_o = 21.48\text{ mm}, \quad \theta_{li} = -24.96^{\circ}, \quad \Delta\theta_i = 19.1^{\circ}, \\ \theta_{lo} = 140^{\circ} \text{ and } \Delta\theta_o = 35.78^{\circ}.$$

The performance metrics produced are as follows;

$$\eta_i = 32.53\%, \quad \psi = 0.92 \text{ and } P_{ind} = 4.03\text{ kW} \text{ at a mass flow rate of } 13.95\text{ kg/min}.$$

Figures 4, 5 and 6 show the values obtained for $F(\Theta)$, η_i and ψ respectively during the iterative optimisation procedure. On the other hand, figures 7 and 8 represent the PV-diagram and the Pressure-angle diagram, respectively, for the produced expander. The considerable pressure fall which starts, as shown in figure 8, at $\theta \approx 140^\circ$ signifies exposure to the discharge port.

9. CASE STUDY 2. EXPANDER WITH A CONTROL VALVE

It is now required to improve the performance of the expander designed in Case Study 1 above. For this purpose, a cam-operated inlet control valve, which has been pre-manufactured to open and close periodically every 90° of the crank rotation, is being considered for the expander. Such a valve increases the isentropic efficiency by expanding the fluid in the working chamber instead of being allowed to escape at high pressure through the discharge port. The thermodynamic simulation predicts the following figures for the performance indices for the expander designed above:

$$\eta_i = 56.38\% , \psi = 0.89 \text{ and } P_{ind} = 3.42 \text{ kW at a mass flow rate of } 6.06 \text{ kg/min} .$$

Despite the improvement obtained in the flow rate and isentropic efficiency values, it is obvious that the original expander, which has been designed to operate in a ported mode, will not produce the required amount of power if used with an inlet control valve. As such, the optimisation procedure is employed in an attempt to explore the possibility of modifying the geometric parameters of the expander to raise the power produced to the required level. The constraints and initial values used for Case Study 1 are employed again here for the optimisation procedure. However, a condition, $A_i = 0$ if $\theta \geq \pi / 2$, has been introduced in the procedure. The optimisation procedure yielded the following values for design parameters;

$$L = 66.14 \text{ mm} , b = 0.064 , L_i = 32.9 \text{ mm} , L_o = 38.53 \text{ mm} , \theta_{ii} = -21.67^\circ , \Delta\theta_i = 7.4^\circ , \\ \theta_{io} = 146.47^\circ \text{ and } \Delta\theta_o = 13.27^\circ .$$

The resulting performance metrics are as follows;

$$\eta_i = 58.67\% , \psi = 0.96 \text{ and } P_{ind} = 4.10 \text{ kW at a mass flow rate of } 7.87 \text{ kg/min} .$$

Figures 9, 10 and 11 show the values obtained for $F(\Theta)$, η_i and ψ respectively during the iterative optimisation procedure. Figures 12 and 13 represent the PV-diagram and the Pressure-angle diagram, respectively, for the new expander.

The results obtained in Case Study 2 highlight the importance of taking into account, at the optimisation stage, whether the expander is intended to run with or without an inlet control valve. With such a valve featured in the design, the resulting expander will enjoy a much higher isentropic efficiency and a lower gas flow rate, than those obtainable from a ported expander, even though the required amount of power will be assured. However, with an inlet control valve used, the size and aspect ratio of the expander are likely to change in order to create a geometric balance between allowing a certain amount of fluid in before the valve closes and providing adequate room for the fluid to expand isentropically to the set discharge pressure. The cut-off angle was set in Case Study 2 to a constant value of 90° which caused the swept volume of the expander to increase from 0.18 litres, as calculated for the ported expander, to 0.194 litres. The increase in the rotor chord length is about 40 mm and the corresponding increase in depth is about 52 mm. However, the performance gains brought about by employing an inlet valve outweigh the minor mechanical challenges which may result by the small increase in the expander size.

10. CONCLUSIONS

This paper features geometric and thermodynamic insights into the workings of limaçon gas expanders. A thermodynamic model has been written and solved numerically at incremental values for the crank shaft angular motion. The model was then utilised in an optimisation procedure to obtain the geometric parameters which would maximise the expander performance for specific power and operating constraints. Two numerical examples have been presented to highlight the fact that the expander performance can be greatly improved if a control valve has been fitted to the inlet port. The results of the numerical examples prove the validity of the models presented and their suitability for expander design.

Nomenclature

A_f , A_p , A_i and A_o $\equiv$ areas of port

A , B and C $\equiv$ constants used for optimisation

a_k and C_k $\equiv$ updated values of A and C

b $\equiv$ limaçon aspect ratio (r/L)

C_r $\equiv$ radial clearance of rotor profile

D_{pd} $\equiv$ eqv. diameter on the downstream side of a port

e_c $\equiv$ specific energy in chamber

E_{cyc} $\equiv$ total energy per rotor revolution

f_p $\equiv$ friction factor

f_r and f_s $\equiv$ density derivatives

$F(\Theta)$ $\equiv$ objective function

h_c $\equiv$ specific enthalpy in chamber

h_{pd} and h_{pu} $\equiv$ specific enthalpies

H_i and H_o $\equiv$ total enthalpies

H_r $\equiv$ the rotor depth

j $\equiv$ counter for nested iterations

k $\equiv$ iteration number: optimisation procedure

K $\equiv$ allowable number of iterations

K_p $\equiv$ loss coefficient

L and L_p $\equiv$ lengths

m $\equiv$ midpoint on the limaçon chord

m_c $\equiv$ mass in working chamber

M_{cyc} $\equiv$ total mass flowing per rotor revolution

n $\equiv$ counter on the θ -axis

N_p $\equiv$ a number used to calculate K_p

$P_{ind} \equiv$ indicated power

p_l and $p_t \equiv$ points on the housing limaçon

$\mathbf{P}_l$ and $\mathbf{P}_t \equiv$ position vectors of the rotor apices

P_c and $\bar{P}_c \equiv$ pressures

r and $r_r \equiv$ radii of base circles

$R_e \equiv$ Reynolds number

$\mathbf{R}_l$ and $\mathbf{R}_t \equiv$ position vector of the port edges

$\hat{\mathbf{R}}_l$ and $\hat{\mathbf{R}}_t \equiv$ unit vectors

$s_c \equiv$ specific entropy in chamber

s_l and $s_t \equiv$ two scalars used for port area calculations

$T_c \equiv$ temperature in chamber

U_i , U_o , U_p and $U_{ps} \equiv$ flow velocities

$U_{ps} \equiv$ sound velocity on the downstream side of a port

$V_c \equiv$ Working chamber volume

w_1 , w_2 and $w_3 \equiv$ weighting factors

W , W_l and $W_t \equiv$ widths of port

$x \equiv$ dryness fraction

$\mathbf{XY} \equiv$ a Cartesian frames

$\Delta \equiv$ vector with a binary Bernoulli distribution

$\Delta h_{io} \equiv$ isentropic enthalpy drop across expander

$\Delta \theta_p \equiv$ the port angular width

$\varepsilon \equiv$ error function for nested iterations

$\eta_i \equiv$ isentropic efficiency

$\theta \equiv$ angle rotated by the chord

$\theta_{co} \equiv$ inlet flow cut off angle

θ_l and $\theta_t \equiv$ angular positions of port edges

$\Theta \equiv$ vector for design parameters

$\Theta_{\min}$ and $\Theta_{\max} \equiv$ constraints on the design parameters

μ_{vap} and μ_{liq} and μ_{pd} $\equiv$ viscosities

ρ_c , $\bar{\rho}_c$ and ρ_{pd} $\equiv$ densities

σ $\equiv$ error function used to end cyclical iterations

τ_{sh} $\equiv$ shaft torque due to chamber pressure

ψ $\equiv$ filling factor

ω $\equiv$ angular velocity of rotor

Subscripts

c $\equiv$ denotes "chamber"

i and o $\equiv$ denote ports (inlet and outlet)

id and od $\equiv$ downstream of ports

iu and ou $\equiv$ upstream of ports

l and t $\equiv$ denote edges (leading and trailing)

p $\equiv$ signifies "port"

pd and pu $\equiv$ downstream and upstream of a port

s and ap $\equiv$ indicate seals (side and apex)

su and apu $\equiv$ upstream of seals

superscripts

A and B $\equiv$ Above and below rotor

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