A MODEL FOR ADAPTIVE RESCHEDULING OF FLIGHTS IN EMERGENCIES (MARFE)

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ABSTRACT. Disruptions to commercial airline schedules are frequent and can inflict significant costs. In this paper we continue a line of research initiated by Vranas, Bertsimas and Odoni [15, 16], that aims to develop techniques facilitating rapid return to normal operations whenever disruptions occur. Ground Holding is a technique that has been successfully employed to combat disruptions at North American airports. However, this alone is insufficient to cope with the problem. We develop an adaptive optimization model that allows the implementation of other tactics, such as flight cancellations, airborne holding and diversions. While the approach is generic, our model incorporates features of Sydney airport in Australia, such as a night curfew from 11:00pm to 6:00am. For an actual day when there was a significant capacity drop, we demonstrate that our model clearly outperforms the actions that were initiated by the air traffic controllers at Sydney.

1. Introduction and literature review. The volume of air traffic has increased considerably over the past two decades, while the capacity of systems such as airports and airways has not kept pace. Demand exceeds capacity at many key airports. Furthermore, unanticipated security related events are also likely to lead to de facto capacity drops.

The high degree of competition among airlines is also noteworthy. This has led to a widespread adoption of Operations Research methodologies and — as a direct result — to schedules that are highly optimized. An unintended consequence of this is that flight operations are highly sensitive to perturbations. Disturbances to the flight schedules due to unpredictable circumstances such as bad weather, aircraft malfunction, or security checks can cause chaos at airports and airline operation centres. The recovery from these schedule perturbations can be assessed by any or all of a number of criteria, some of which may be in conflict. Flights may need to be delayed, diverted or cancelled, causing inconvenience to passengers and reducing airline profits.


Key words and phrases. Air traffic management, Adaptive flight rescheduling, Discrete Optimization, Integer Programming.
Given that the demand for arrivals at an airport will, at times, exceed capacity, it is beneficial to delay some of these flights at their originating airport. This is because delays of aircraft on the ground are cheaper and safer than equivalent delays in the air. Ground holding has been used extensively in the United States since the air traffic controllers’ strike of 1981, and is managed by the Air Traffic Control System Command Center (ATCSCC). In Australia, however, ground holding seems to be a relatively new strategy, used mainly at Sydney’s Kingsford Smith Airport, and supported by the automated CTMS (Central Traffic Management System)\[1\].

In [13], a stochastic programming model for a single airport is considered. In this model capacity profiles are considered to be random and it is assumed that a probability distribution on these capacity scenarios is known. In [14], two models are formulated and analyzed. The first, an integer programming model, is deterministic. The objective is to minimize the total ground holding costs of all flights scheduled to arrive in the time horizon selected by the study. It assumes that airport capacities and flight duration times are known in advance with certainty and that airborne holding is always more expensive than ground holding. The second model is stochastic, and some airborne holding may occur because of uncertainty of airport capacity over the subsequent few hours. A number of airport capacity scenarios are possible, and the probabilities of the scenarios are assumed to be static.

In [16], Vranas, Bertsimas and Odoni provide the first model that considers this problem in a dynamic environment that allows for weather changes, and aircraft becoming available or unavailable. The model provides updated ground holding decisions and its objective minimizes the sum of airborne delay and ground delay.

Bertsimas and Patterson [2] consider the Traffic Flow Management Problem (TFMP) caused by disturbances to flight schedules. This includes determination of aircraft release times at airports (ground holding) and optimal aircraft speeds while airborne. Their model is an integer program that considers the capacities of the en route airspace, airport capacities at different time intervals, and cost per unit time of holding aircraft on the ground and in the air. The objective function minimizes the total delay cost.

Navazio and Romanin-Jacur [10] construct an integer programming model to analyze multi-airport ground holding. Their model minimizes the overall delay cost subject to airport capacity, connections, and time constraints imposed by airlines. The model distributes the ground holding delays among a set of flights originating at a set of airports. Brunetta, Guastalla and Navazio [3] improve a heuristic given in [10].

Hoffman and Ball [8] present five different models of the Single Airport Ground Holding problem in the presence of banking constraints to accommodate the hubbing operations of major airlines in the United States.

In [12], Rosenberger, Johnson and Nemhauser consider the disruption recovery problem from an airline’s point of view. The recovery is carried out in stages. The first stage reroutes aircraft, delays flight legs or cancels them. Subsequent stages perform recovery of crew and passengers. The first of the three stages (aircraft recovery) is modelled as a set-packing problem.

1.1. Contributions of this paper. In this paper we view the flight schedule disruption problem from the “common good” perspective. That is, we begin with a premise that — with a judicious choice of interventions (e.g., ground holding, cancellations and diversions) — it is possible to minimize the detrimental effects of schedule disruptions. Our belief is that by minimizing a suitably constructed cost
function of these effects, it is possible to significantly reduce recovery costs to all the key participants: passengers, airlines, airport corporations and air traffic regulatory agencies.

To demonstrate the feasibility of the above approach we develop a new optimization model called MARFE (Model for Adaptive Rescheduling of Flights in Emergencies). An important conceptual advance of MARFE is that it is an adaptive model that determines optimally modified schedules both for aircraft that are still on the ground and for those that are already airborne. Consequently, MARFE enables us to optimally adapt to reduced airport capacity levels, almost as soon as a warning of an impending capacity emergency is issued. The logical structure of MARFE can be seen from the flow chart in Figure 1.

**Figure 1.** MARFE: Model for Adaptive Rescheduling of Flights in Emergencies
MARFE has been developed with realistic capacity reduction scenarios at Sydney airport in mind, and has been validated for that airport using both synthetic and real data. The results of this validation indicate that delay reductions of the order of 30-50% are possible with the help of rescheduling that optimizes the recovery process.

The description of the MARFE model, a theoretical justification of its structure, and empirical validation of its effectiveness constitute the bulk of this paper. In the remainder of this introduction we briefly mention the existing optimization models that address related problems. This will help the reader place MARFE in its proper context in the still evolving literature of the subject. For a more detailed survey of airport recovery literature we refer the reader to Filar, Manyem and White [5].

Our MARFE model can be viewed as descending from the models of [2] and [10], and as an extension of an intermediate ground holding model reported by us in [4]. Its main distinguishing features are the multistage, adaptive structure, and the incorporation of the many realistic constraints that apply at an airport such as Sydney.

2. The model — MARFE. The model applies to a network of capacitated airports, although in practice we generally concentrate on part of the network by relaxing capacity restrictions at other airports (nodes of the network). Of course a single airport is part of the network in this context.

As in [10], only arrival capacities are used. Gilbo [6, 7] has studied the interaction between arrival and departure capacities at airports. Generally, though, arrival capacities are more restrictive than departure capacities. In bad weather, an airport can handle fewer arrivals than departures.

A key feature of MARFE and its predecessor models is the discretisation of time into periods or intervals. There could be any fixed number of these periods, and they could be uniform or varying in length throughout the day. Most commonly we will deal with a 24 hour day divided into 96 periods of 15 minutes each.

The model has two distinct modes of application, referred to as stages. Stage one is an enhancement of the Navazio and Romanin-Jacur (NRJ) model [10]. It is run some time before the day of operations (usually the previous evening), using current expectations of flight schedules and airport capacities. At this time few of the flights involved have yet commenced, and so it is quite feasible to impose ground delays or to cancel flights as necessary.

Stage two is more of a departure from the NRJ model. It is run as and when required on the day of operations, whenever reductions in airport capacity become apparent. Since some flights are now airborne, it may be necessary to impose airborne holding or even to divert flights to alternate airports.

Stage two can be applied multiple times, but stage one would normally only be applied once, for a given day. These two modes of application are combined seamlessly in the one model, as described below.

The repeated application of MARFE generates a sequence of feasible schedules, numbered consecutively from 0, such that schedule \( i - 1 \) is input to the model when it is required to compute schedule \( i \). Thus the original published schedule is schedule 0, and the stage one application of MARFE with assumed capacities for the following day produces schedule 1. The first application of stage two produces schedule 2.
**Assumptions:** (1) When creating each schedule, we assume that the capacity forecast is perfect for the remainder of the planning period. (2) Cancellations are irreversible — a flight cancelled in schedule $i$ remains cancelled in all subsequent schedules. (3) Similarly, delays are also irreversible — the delay suffered by a flight in schedule $i$ cannot be recovered in any subsequent schedules. (4) The diversion airport always has sufficient capacity to receive diverted flights, both in the surrounding airspace and on the ground (in the form of taxiways and gates).

In this process some parameters are persistent — they are the same for all schedules (e.g. curfew times). Other parameters are schedule specific (e.g. airport capacity). Yet other entities are variables with values from an output of schedule $i - 1$ that become parameter inputs to schedule $i$ (e.g. the computed arrival time of a flight becomes the planned arrival time of that flight in the next schedule).

The reduction in capacity that triggers an application of stage 2 of MARFE is referred to as a capacity emergency. The restrictions of the $i^{th}$ such emergency begin in time period $t^i_1$ and last until time period $t^i_2$. Rescheduling to mitigate the effects of the emergency begins in time period $t^i_0$. Generally we assume that $t^i_0 \leq t^i_1$ for all $i \geq 1$, although we experiment with a relaxation of this condition in Section K.

Flights that have not yet left their airport of origin in time period $t^i_0$ may be ground held or cancelled. Usually the first 15 minutes of ground holding delay is relatively inconsequential, so we allow the cost coefficient of this first period to differ from that of subsequent periods. In principle, this idea could be extended to having different costs for each of the first few periods of ground holding, with a corresponding increase in the number of variables and constraints for each flight.

Any flight that is en route in period $t^i_0$ may be subject to airborne holding or diversion. The total airborne delay accumulates from one schedule to the next. Each flight has a unique diversion airport, which will be used if the anticipated total amount of airborne holding delay exceeds a previously determined limit. It is assumed that a diverted flight dwells at its diversion airport for at least one time period, and that it experiences no airborne delay on its eventual journey to its destination.

Each flight may have any number of successor flights. In particular, a following flight that uses the same aircraft is a successor, but other flights that involve the same flight crew or that continue the journeys of a significant group of passengers can also be included in the successor set. Each successor flight will inherit delay from its predecessor if there is insufficient slack time between the two flights to absorb it. Successors of a cancelled flight will also be cancelled.

Finally, there is a (punitive) cost for any flight that arrives during any curfew that may be mandated at its destination airport. For this purpose we introduce parameters for the first and last periods of non-curfew operations at each airport. For example, if the curfew ends at 6 am and 15 minute time periods are being used, then the first period of non-curfew operations would be number 25. If there is no curfew at a particular airport then the first period of non-curfew operations would be number 1.

### 2.1. A summary of the model

Presented below is an outline of the model in the $i^{th}$ stage:

**Objective:** Minimize the sum of delay costs, curfew violation costs, cancellation costs, and diversion costs, over all flights arriving at all airports.

**Constraints:**
1. Constraints on airport capacities, for each time interval,
2. Constraints that compute the ground holding and airborne holding delays, and impose a bound on such delays,
3. Constraints that determine whether a flight is to
   • be cancelled (these are coupled with the ground delay constraints above),
   • be diverted (coupled with the airborne delay constraints above), and
   • violate the curfew (coupled with ground and airborne delays),
4. Constraints that link arriving flights to departing successor flights, and
5. Integrality constraints.

Within each set of constraints, there are those that link each schedule $i$ with the previous schedule $i-1$.

**Remark:** The remainder of this section is devoted to a detailed description of the model and to a rigorous verification of its validity. Consequently, readers interested primarily in the interpretation and qualitative results may wish to proceed directly to Section 3.

### 2.2. Notation.
Following is the notation used in MARFE. It is based on the notation of the NRJ model [10]. Since the number of entities introduced is large we group them into several cognate categories.

**Parameters that remain constant in all schedules**

- $T$ set of time periods.
- $t$ a time period, $t \in T$.
- $Z$ set of airports.
- $z$ an airport, $z \in Z$.
- $F$ set of flights.
- $f$ a flight, $f \in F$.
- $r_0^f$ published arrival period of flight $f$ ($r_0^f \in T$).
- $T_f$ set of time intervals in which flight $f$ may land at its destination airport. Since flight diversion is permitted, $T_f = \{r_0^f, \ldots, |T|\}$.
- $K_{z,t}^0$ initial (input to stage 1) forecast of arrival capacity of airport $z$ for time period $t$. Used to compute schedule 1.
- $F_z$ set of flights arriving at airport $z$. For example, $F_{\text{syd}} = \text{set of flights that arrive at Sydney}$.
- $F_{z,d}^f$ set of flights whose diversion airport is $z$.
- $S_f$ set of successors of flight $f$.
- $c_1^f$ cost of the first period of ground delay for flight $f$.
- $c_2^f$ cost of the second and each subsequent period of ground delay for flight $f$. $c_2^f \geq c_1^f$.
- $c_{aj}$ cost per period of airborne delay of flight $f$. Generally, $c_{aj}^f < c_{aj}^f$ for $j = 1, 2$; ground holding delay is preferable to airborne delay.
- $\Delta_{max}$ maximum allowed total delay (ground and air combined) for any flight that is not diverted.
- $\Delta_{max}^a$ maximum allowed airborne delay for any flight.
k_f  cancellation cost of f.
\( w_z^1 \)  first time interval of non-curfew operations at airport z.
\( w_z^2 \)  last interval of non-curfew operations at airport z.
\( e_z \)  curfew breaking cost for each flight at airport z.
\( \alpha_f \)  flying time of flight f.
\( \phi_{f,g} \) minimum number of time intervals required between arrival of flight f and departure of its successor g, (also known as f-g turnaround time)
\( \Phi_{f,g} \)  the service time between f and g, including the flight time of g, which equals \( \phi_{f,g} + \alpha_g \)
\( \alpha'_f \) extra flying time of flight f to its diversion airport.
\( \alpha^r_f \)  flying time of flight f from its diversion airport to its destination.

Parameters that change depending on a schedule i

\( r^i_f \) revised arrival period of flight f computed in schedule i \( \equiv \) planned arrival period of flight f input to schedule \( i + 1 \).
\( t^i_0 \)  the first time period during which action is taken in schedule \( i + 1 \) to minimize the effects of the \( i^{th} \) capacity emergency, \( i \geq 1 \).
\( t^i_1 \)  the first time period of capacity emergency i.
\( t^i_2 \)  the first time period of restored capacity, after emergency i.
\( K^i_{z,t} \) for \( i \geq 1 \); revised arrival capacity of airport z at period t, for \( i^{th} \) application of stage 2 to produce schedule \( i + 1 \). Differs from the previous \( K^{i-1}_{z,t} \) only in periods \( t^i_1, \ldots, t^i_2 - 1 \).
\( \sigma^i_{f,g} \) in schedule i, the number of time periods of slack between flight f and its successor g, given by the difference between the planned departure period of g and the planned arrival period of f, minus the necessary service time. Also given by \( r^i_g - \alpha_g - r^i_f - \phi_{f,g} \).
\( F^i_{z,A} \)  set of flights bound for airport z that, according to schedule i, are still in progress (either airborne or grounded at their diversion airport) in time period \( t^i_0 \).
\( F^i_{z,G} \)  set of flights bound for airport z that have not yet departed (according to schedule i) at the beginning of time \( t^i_0 \).
\( F^i \) union over all airports \( z \in Z \) of \( F^i_{z,A} \) and \( F^i_{z,G} \), being the set of flights of interest when schedule \( i + 1 \) is computed. All other flights have already completed their journeys at time \( t^i_0 \), and so are irrelevant.

Decision variables

\[
x^i_{f,t} = \begin{cases} 
1 & \text{if flight } f \text{ arrives at its destination in or before time period } t \text{ in schedule } i, \\
0 & \text{otherwise.}
\end{cases}
\]
For notational and visual convenience, the model also contains a number of variables that are functions of the preceding decision variables. We refer to these as consequentially determined variables.

**Consequentially determined variables**

- $\Delta^{g,i}_f$: ground delay imposed by schedule $i$ on flight $f$.
- $\Delta^{a,i}_f$: airborne delay imposed by schedule $i$ on flight $f$ (occurs only at destination airport).
- $\Omega^i_f$: airborne delay of flight $f$ accumulated over schedules up to and including schedule $i$. $\Omega^0_f = 0$.
- $\Delta^{e,i}_f$: en route delay imposed by schedule $i$ on flight $f$. (Comprises airborne holding and diversion delays.)
- $\Delta^{d,i}_f$: ground delay imposed by schedule $i$ on flight $f$ at its diversion airport.
- $\gamma^i_f$: ground delay cost in schedule $i$ of flight $f$ at its airport of origin.

\[
\delta_i^f = \begin{cases} 
1 & \text{if flight } f \text{ experiences an accumulated delay of at least one time period up to and including schedule } i, \text{ (that is, if } r^i_f - r^0_f \geq 1), \\
0 & \text{otherwise.}
\end{cases}
\]

\[
\xi_i^f = \begin{cases} 
1 & \text{if flight } f \text{ arrives in schedule } i \\
0 & \text{otherwise.}
\end{cases}
\]

\[
\beta_i^f = \begin{cases} 
1 & \text{if } f \text{ is diverted in schedule } i, \\
0 & \text{otherwise.}
\end{cases}
\]

\[
B_i^j = \begin{cases} 
1 & \text{if } f \text{ is diverted in schedule } j \text{ for any } j \leq i, \\
0 & \text{otherwise.}
\end{cases}
\]

(Note that $\delta^0_f = 0$, $\xi^i_f = x^i_{f,T_i}$, $\beta^0_f = 0$ and $B^0_f = 0$.)

\[
x_{d,i,f,t} = \begin{cases} 
1 & \text{if } f \text{ arrives at its diversion airport in or before period } t \text{ in schedule } i, \\
0 & \text{otherwise.}
\end{cases}
\]

\[
b_i^f = \begin{cases} 
1 & \text{if flight } f \text{ breaks the curfew at its arrival airport in schedule } i, \\
0 & \text{otherwise.}
\end{cases}
\]

$a_{1,i}^f$, $a_{2,i}^f$, $b_{1,i}^f$, $b_{2,i}^f$ and $h_i^f$ are auxiliary variables for flight $f$ in schedule $i$, and they operate as follows:

It is intended that, as a consequence of the constraints of the model given below, the auxiliary decision variable $a_{1,i}^f$ be negative, and $b_{1,i}^f = 1$, if and only if flight $f$ arrives at its destination airport $z$ sufficiently early in the morning (that is, before $w_z^1$) to break the curfew. Similarly, $a_{2,i}^f$ should be negative, and hence $b_{2,i}^f = 1$, if and only if $f$ arrives after the evening curfew time (that is, after $w_z^2$).
Therefore, in the case of a curfew violation by flight $f$, exactly one of the two variables $a_{j}^{1,i}$ and $a_{j}^{2,i}$ should be negative. One of $b_{j}^{1,i}$ and $b_{j}^{2,i}$ should be one and the other zero, and consequently $b_{j}^{2} = 1$.

Note that if flight $f$ is cancelled in schedule $i$, that is if $\xi_{j}^{i} = 0$, then all three variables, $b_{j}^{1,i}$, $b_{j}^{2,i}$ and $b_{j}^{2}$ should be zero.

The value of $b_{j}^{2}$ is to be 1 if and only if flight $f$ experiences its first period of ground hold delay in schedule $i$. This first period has a cost of $c_{j}^{1}$, whereas subsequent periods each cost $c_{j}^{2}$ (see Lemma 4 and Theorem 4 below).

2.3. Objective function. There are many reasonable choices for the objective function. The NRJ model uses a simple minimization of weighted ground holding delay. Here we minimize the total incremental cost of a schedule, given the conditions implied by the previously computed schedule. Mathematically, our objective has the following form.

$$
\text{minimize } \sum_{z \in Z} \sum_{f \in F_{z,G}^{i-1}} \left[ \gamma_{j}^{i} f + k_{j}(\xi_{j}^{i-1} - \xi_{j}^{i}) + e_{z}(b_{j}^{1,i} - b_{j}^{2,i}) \right] + \\
\sum_{z \in Z} \sum_{f \in F_{z,A}^{i-1}} \left[ c_{j}^{z}(\Delta_{j}^{z,i} - \Delta_{j}^{z,i}) + e_{z}(b_{j}^{1,i} - b_{j}^{2,i}) + c_{j}^{2}\Delta_{j}^{d,i} \right] 
$$

(1)

The first double summation captures the incremental cost incurred by flights that are still on the ground in period $t_{0}^{i-1}$, the time at which action is taken to respond to the disruption of schedule $i - 1$. The second double summation captures the incremental cost incurred by flights that are already airborne at $t_{0}^{i-1}$.

Note that the total additional delay experienced by a flight $f$ in schedule $i$ is simply $r_{j}^{i} - r_{j}^{i-1}$, where $r_{j}^{i} = \sum_{t \in T_{j}} t(x_{f,t}^{i} - x_{f,t-1}^{i})$ is the scheduled arrival time of $f$ in schedule $i$. For any $z$, this delay will be entirely a ground holding delay for flights in $F_{z,G}^{i-1}$, and en route (airborne holding and diversion) delay for flights in $F_{z,A}^{i-1}$. The constraints presented below (e.g., see (12), (13) and (20)) ensure that the cost of this additional delay is properly distributed among the terms of the objective function.

Cancellation of flight $f$ in schedule $i$ is indicated by $\xi_{j}^{i-1} - \xi_{j}^{i}$. This difference will have value 1 if and only if flight $f$ does arrive in schedule $i - 1$ but doesn’t in schedule $i$. Since only flights in $F_{z,G}^{i-1}$ can be cancelled, the corresponding costs are properly covered by the second term in the first double summation of the objective. Note that $r_{j}^{i} = 0$ if flight $f$ is cancelled in schedule $j$ for any $j \leq i$.

On the other hand, the curfew can be violated by flights in both $F_{z,G}^{i-1}$ and $F_{z,A}^{i-1}$. Changes to curfew violation status of a flight between schedules $i - 1$ and $i$ can be detected by examination of $b_{j}^{1,i} - b_{j}^{2,i}$. Note, however, that it is possible for flight $f$ to break the curfew in schedule $i - 1$, but not break it in schedule $i$. This could happen if, for example, the flight were cancelled in schedule $i$. Therefore, the difference $b_{j}^{1,i} - b_{j}^{2,i}$ can take any of the values in $\{-1, 0, 1\}$.

2.4. Constraints. Due to the many realistic features that have been incorporated into MARFE, the set of constraints is considerably more complex than those in the predecessor NRJ model [10]. To make this subsection more transparent, these constraints have been grouped according to the main function that they perform in the model. However, a reader needs to be aware that — because of the coupling between the model’s components — these groupings are necessarily overlapping. The
constraints are explained in more detail in the next section (Validity of MARFE), in the proofs to Lemma [1] and Theorem [1]. Unless specified otherwise, each of these constraints applies to each flight $f$ in $F^{i-1}$.

**Capacity Constraints**

\[
\sum_{f \in F_z \cap F^{i-1}} (x^{i}_{f,t} - x^{i}_{f,t-1}) + \sum_{f \in F_z' \cap F^{i-1}} (x^{d,i}_{f,t} - x^{d,i}_{f,t-1}) \leq K^{i-1}_{z,t} \quad \forall \ z \in Z, \ \forall \ t \in \{t_0^{-1}, \ldots, |T|\}
\]

(2)

**Constraints related to arrival times**

\[
x^{i}_{f,t} = 0 \quad \forall \ t \in \{1, \ldots, r^{i-1}_{f} - 1\}
\]

(3)

\[
x^{i}_{f,t} - x^{i}_{f,t-1} \geq 0 \quad \forall \ t \in T_f
\]

(4)

\[
r^{i}_f = \sum_{t \in T_f} t(x^{i}_{f,t} - x^{i}_{f,t-1})
\]

(5)

\[
x^{d,i}_{f,t} = 0 \quad \forall \ t \in \{1, \ldots, r^{i-1}_{f} - 1\}
\]

(6)

\[
x^{d,i}_{f,t} - x^{d,i}_{f,t-1} \geq 0 \quad \forall \ t \in T_f
\]

(7)

\[
r^{d,i}_f = \sum_{t \in T} t(x^{d,i}_{f,t} - x^{d,i}_{f,t-1})
\]

(8)

( = 0 if $f$ is not diverted in schedule $i$)

\[
\xi^{i}_{f} = x^{i}_{f,|T|}
\]

(9)

\[
\xi^{i}_{f} \leq \xi^{i-1}_{f}
\]

(10)

\[
\xi^{i}_{f} = 1 \quad \forall \ f \in F^{i-1}_{z,A}, \ \forall \ z \in Z
\]

(11)

**Constraints related to delays**

\[
\Delta^{g,i}_f + \Delta^{c,i}_f = r^{i}_f - r^{i-1}_f \xi^{i}_f
\]

(12)

\[
\gamma^{i}_f = c^{2}_f \Delta^{g,i}_f - (c^{2}_f - c^{1}_f)h^{i}_f
\]

(13)

\[
h^{i}_f \leq 1 - \delta^{i-1}_f
\]

(14)

\[
h^{i}_f \leq \Delta^{g,i}_f
\]

(15)

\[
Mh^{i}_f \geq \Delta^{g,i}_f - M\delta^{i-1}_f
\]

(16)

\[
M\delta^{i}_f \geq r^{i}_f - r^{0}_{f} \xi^{i}_f
\]

(17)

\[
\delta^{i}_f \leq r^{i}_f - r^{0}_{f} \xi^{i}_f
\]

(18)

\[
\Delta^{g,i}_f \geq 0
\]

(19)
\[ \Delta_{f}^e,i = (\alpha_d^f + \alpha_r^f)\beta_f^i + \Delta_{f}^{d,i} + \Delta_{f}^{a,i} \quad \forall \ f \in F_{z,A}^{i-1} \forall \ z \in Z \]  

\[ \Delta_{f}^e,i = 0 \quad \forall \ f \in F_{z,G}^{i-1} \forall \ z \in Z \]  

\[ \Delta_{f}^g,i = 0 \quad \forall \ f \in F_{z,A}^{i-1} \forall \ z \in Z \]  

\[ \Delta_{f}^{a,i} \geq 0 \]  

\[ \Omega_f^i = \Omega_{f}^{i-1} + \Delta_{f}^{a,i} \]  

\[ \Omega_f^i \leq \Delta_{max}^a \]  

**Constraints related to diversions**

\[ \Delta_{f}^{d,i} \geq \beta_f^i \]  

\[ \Delta_{f}^{d,i} \leq M B_f^i \]  

\[ B_f^i = B_f^{i-1} + \beta_f^i \]  

\[ r_{f}^{d,i} = r_{f}^{d,i-1} B_f^{i-1} + (r_f^{i-1} + \alpha_f^d)\beta_f^i \quad \forall \ f \in F_{z,A}^{i-1} \forall \ z \in Z \]  

\[ r_{f}^{d,i} = 0 \quad \forall \ f \in F_{z,G}^{i-1} \forall \ z \in Z \]  

\[ B_f^i = 0 \quad \forall \ f \in F_{z,G}^{i-1} \forall \ z \in Z \]  

\[ x_{f,t}^i \geq \xi_f^i - B_f^i \forall \ t \in \{r_f^0 + \Delta_{max}, \ldots, |T|\}, \]  

**Coupling constraints**

\[ x_{f,t}^i - x_{g,u}^i \geq 0 \quad \forall \ t \in T_f, \forall \ g \in S_f, \forall \ f \in F \]  

and \ \forall \ u \in T_g \ such \ that \ 

\[ u = t + \alpha_g + \phi_{f,g} (= r_g^{i-1} + (t - r_f^{i-1}) - \sigma_{f,g}^{i-1}) \]  

**Constraints related to curfew violations**

\[ a_{f}^{1,i} = r_f^i - w_z^1 \]  

\[ a_{f}^{2,i} = w_z^2 - r_f^i \]  

\[ -a_{f}^{1,i} \leq M b_f^{1,i} \]  

\[ a_{f}^{1,i} \leq M(1 - b_f^{1,i}) - 1 \]  

\[ -a_{f}^{2,i} \leq M b_f^{2,i} \]  

\[ a_{f}^{2,i} \leq M(1 - b_f^{2,i}) - 1 \]  

\[ b_f^i \geq b_f^{1,i} + \xi_f^i - 1 \]
Proof.

c_{ground holding delay has cost appropriate values of input parameters, including some that are calculated in schedule

If flight \( f \) is a large positive constant. (48)

We commence by considering the following result concerning the calculation of

Validity of MARFE. Here we give a formal proof that a feasible solution to MARFE will take account of all relevant costs as intended. These costs are as described in the statement of Theorem 1 below. Note that we regard the \( x^i_j \) as the main decision variables, and claim that the values of the other variables in the model are consequentially determined. Each feasible solution to \( 4 \) can thus be characterised by a vector \( \psi^i = (x^i_{f,t})_{f \in F, t \in T} \).

We commence by considering the following result concerning the calculation of the ground holding cost in any feasible solution. Note that we need only consider here flights that are in \( F^i_{z,G} \) for some airport \( z \), since these are the only flights that are permitted to experience ground holding delay. Indeed the objective function accounts for ground holding delay only for such flights. Also, it is apparent that if \( f \in F^i_{z,G} \) then \( f \in F^j_{z,G} \) for all \( j \leq i - 1 \).

Lemma 1. Let \( \psi^i = (x^i_{f,t})_{f \in F, t \in T} \) be a feasible solution to the model, with appropriate values of input parameters, including some that are calculated in schedule \( i - 1 \).

If flight \( f \) is in \( F^i_{z,G} \) for some airport \( z \) experiences its first period of ground holding delay in schedule \( i \) then the constraints of MARFE ensure that the ground holding cost \( \gamma^i_f = c^1_f + c^2_f(\Delta^i_{f} - 1) \). Otherwise, \( \gamma^i_f = c^3_f(\Delta^i_{f} - 1) \). That is, the first period of ground holding delay has cost \( c^1_f \) while subsequent periods have cost \( c^3_f \) each.

Proof. 1. Cancellation of flight \( f \) in schedule \( i \) (or earlier) is indicated by \( x^i_{f,t} = 0 \) for all \( t \in T \). In this case, \( r^i_f = \xi^i_f = 0 \) by (13) and (14), and so \( \Delta^i_{f} = 0 \) by (12) and (11), since \( f \in F^i_{z,G} \) for some airport \( z \). Hence \( h^i_f = 0 \) by (15) and (16) and \( \gamma^i_f = 0 \) by (13).

2. If flight \( f \) arrives in schedule \( i \), then \( 10 \) ensures that \( \xi^i_f = 1 \). By (14) and the non-negativity of delays, \( r^i_f \geq r^{i-1}_f \), and so, recursively, \( r^i_f \geq r^{0}_f \) for all \( j \leq i \).

The schedule \( i \) input parameter \( \delta^i_f \) has value 1 if and only if flight \( f \) has already experienced nonzero ground holding delay in schedule \( i - 1 \) or previously. This value will be fixed by inequalities (17) and (15) when schedule \( i - 1 \) is considered, since flight \( f \) has been ground delayed and not cancelled if and only if \( \delta^i_{f-1} = 1 \) and \( r^{i-1}_f - r^0_f \geq 1 \).
(a) If flight $f$ has already experienced ground hold delay before schedule $i$ then $h_f^i = 0$ by (14). Alternatively if flight $f$ experiences no ground hold delay in schedule $i$ then $\Delta_{f}^{i} = 0$ and $h_f^i = 0$ by (15). In either case, (14) is satisfied, and $\gamma_f^i = c_f^i \Delta_{f}^{i}$ by (13). That is, any periods of ground hold delay are second or subsequent periods and are costed as such.

(b) If flight $f$ experiences its first period of ground delay in schedule $i$ then $\delta_f^i = 0$ and $\Delta_{f}^{i} \geq 1$, so (10) will ensure that $h_f^i = 1$, and (13) and (14) are both satisfied. In this case (13) becomes $\gamma_f^i = c_f^i \Delta_{f}^{i} - (c_f^i - c_f^i) = c_f^i + c_f^i (\Delta_{f}^{i} - 1)$.

We now proceed to the main consistency theorem.

**Theorem 1.** Let $\psi^i = (x_{i,j}^i)_{f \in F, i \in T}$ be a feasible solution to (2-48). The following apply to $\psi^i$, for each flight $f$:

1. If $\xi_f^i = 0$ then the flight is cancelled and its arrival time is meaningless. Such a flight is a member of set $F_{z,G}^{-1}$ for some airport $z$. It incurs cancellation cost in schedule $i$ if it was not previously cancelled, but it incurs no delay cost. It will not violate curfew in schedule $i$ and so incurs no curfew penalty, but it may reverse a curfew penalty from the previous schedule. The flight will remain cancelled in later schedules in which it is considered, and has no diversion implications in any schedule. Explicitly, if $\xi_f^i = 0$ then $\xi_f^i = 0$, $\gamma_f^i = 0$, $b_f^i = 0$, $r_f^j = 0$, and $f \in F_{z,G}^{-1}$ for all $j \geq i$ such that $f \in F_{z,G}^{-1}$.
   Each successor flight (member of $S_f$) is also cancelled in schedule $i$.
2. If $\xi_f^i = 1$ and $B_f^i = 0$ then the flight is not cancelled and not diverted. If the flight’s arrival time is later than that previously scheduled then a delay cost is incurred, at a ground hold cost rate if $f \in F_{z,G}^{-1}$ and at an airborne hold cost rate if $f \in F_{z,A}^{-1}$. The flight incurs a curfew penalty if and only if its arrival time falls within the curfew hours at its destination $z$. The flight incurs no cancellation cost.
3. If $\beta_f^i = 1$ (the flight is freshly diverted in schedule $i$), then the flight is not cancelled. The delay experienced by the flight in schedule $i$ is $\alpha_f^i + \Delta_f^{d,i} + \alpha_f^i$, that is, the sum of the flying time to the diversion airport, the anticipated time on the ground at that airport, and the flying time of the recovery flight to the destination airport — of these three components, only one of them ($\Delta_f^{d,i}$) is incremental. The incremental cost comprises the total flying time at airborne cost, the ground time at the higher of the two ground hold cost rates ($c_f^i$), and any curfew penalty that may be incurred.
4. If $B_f^{i-1} = 1$, then the flight has been diverted in a previous schedule. The flight is not cancelled, and will not be diverted again ($\beta_f^i = 0$). The only cost related to this flight in the objective function in schedule $i$ is the incremental ground holding cost of $f$ at the diversion airport, $c_f^i \Delta_f^{d,i}$, and curfew penalty if applicable.

**Proof.** 1. If $\xi_f^i = 0$ then $f \in F_{z,G}^{-1}$ for some airport $z$, otherwise (11) would be violated. Flight $f$ was not cancelled in previous schedules if and only if $\xi_f^i = 1$, in which case $f$ incurs cancellation cost in schedule $i$.

By (10), $\xi_f^i = 0$ and so $f \in F_{z,G}^{-1}$ for all $j \geq i$ for which flight $f \in F_{z,G}^{-1}$.
Now, for all such \( j; x_{f,t}^j = 0 \) by (13), and \( x_{f,t}^i = 0 \) for all \( t \) by (3) and (4).

Therefore, as in the proof of Lemma 1, \( r_f^j = 0 \), \( \Delta_f^{j,i} = 0 \), and \( \gamma_f^j = 0 \). Moreover, \( b_f^j = 0 \) by (14). Note that inequalities (41) and (42) are satisfied since \( \xi_f^j = 0 \). Observe that \( r_f^{d,j} = B_f^j = 0 \) by (31) and (32), and so \( x_{f,t}^{d,j} = 0 \) for all \( t \in T \) by (4), (7) and (8). If \( b_f^{j-1} = 0 \) then there is no curfew penalty contribution in schedule \( i \). Alternatively, if \( b_f^{j-1} = 1 \), then schedule \( i \) incurs a cost of \(-e_z\), indicating that a curfew penalty previously anticipated will no longer need to be imposed because the curfew violation can be avoided in schedule \( i \). By (34), \( x_{g,u}^i = 0 \) for all \( g \in S_f \) and all \( u \in T_g \), so successors of \( f \) are also cancelled.

2. Consider an arbitrary flight \( f \) for which \( \xi_f^i = 1 \) and \( B_f^i = 0 \).

By (10), \( \xi_f^{i-1} = 1 \) also, and hence it is clear that no cancellation cost is incurred by \( f \) in the objective function (11). The incremental arrival delay experienced by flight \( f \) in schedule \( i \) is \( \Delta_f^{g,i} + \Delta_f^{c,i} \), the difference between the new \( (r_f^j) \) and old \( (r_f^{j-1}) \) scheduled arrival times, as given by (12).

For the case of \( f \) being in \( F_{z,G}^{i-1} \) for some \( z \), (21) yields \( \Delta_f^{c,i} = 0 \) and the delay is costed in the objective function as ground holding and is correct by Lemma 1. Alternatively, if \( f \in F_{z,A}^{i-1} \) for some \( z \), then \( \Delta_f^{g,i} = 0 \) by (22). Since \( B_f^i = 0, \beta_f^i = \Delta_f^{d,i} = 0 \) from (26) and (27). Hence \( \Delta_f^{a,i} = \Delta_f^{a,i} \) by (20), and all the delay is costed as airborne.

In either case, now suppose that flight \( f \in F_z \) obeys the curfew (if any) at its arrival airport \( z \) in schedule \( i \). Thus \( w_1^z \leq r_f^1 \leq w_2^z \), and \( a_f^{1,i} \) and \( a_f^{2,i} \) are both nonnegative by equations (35) and (36) respectively. By (35) and (40), \( b_f^{1,i} = b_f^{2,i} = 0 \) and so \( b_f^i = 0 \) by (18).

Conversely, suppose that \( f \) violates the morning or evening curfew. Then \( r_f^j < w_1^z \) or \( r_f^j > w_2^z \) and so either \( a_f^{1,i} \) or \( a_f^{2,i} \) is negative. Thus, either \( b_f^{1,i} = 1 \) or \( b_f^{2,i} = 1 \), so \( b_f^i = 1 \) by (41) or (29).

Hence \( b_f^i = 1 \) if and only if \( f \) violates the curfew.

The curfew penalty \( e_z \) is imposed upon flight \( f \) in schedule \( i \) if \( b_f^{j-1} = 0 \) and \( b_f^i = 1 \). That is, the curfew penalty is incurred if \( f \) violates curfew in schedule \( i \) but not in schedule \( i - 1 \). If \( b_f^i = b_f^{j-1} = 0 \) then there is no change of curfew status for flight \( f \) between schedules \( i - 1 \) and \( i \) so no incremental curfew cost is incurred. Alternatively, if \( b_f^{j-1} = 1 \) and \( b_f^i = 0 \), then schedule \( i \) incurs a cost of \(-e_z\) as seen above.

3. If \( \beta_f^i = 1 \), then it follows from equation (29) that \( B_f^j = 1 \) and \( B_f^{i-1} = 0 \) since all three variables are binary. That is, the flight is not diverted in any schedule \( j \) where \( j \leq i - 1 \), and is diverted in the current schedule \( i \).

Since \( B_f^j = 1 \), from (32), it follows that \( f \) cannot be in \( F_{z,G}^{i-1} \), hence \( f \in F_{z,A}^{i-1} \) and \( f \) is not cancelled. Now \( \Delta_f^{a,i} = 0 \) by (28) and so (30) yields \( r_f^{d,i} = r_f^{i-1} + \alpha_f^i \). Consequently the incremental delay given by (20) is \( \Delta_f^{a,i} = \alpha_f^i + \gamma_f^i + \Delta_f^{d,i} \), with cost \( c_f^e(\Delta_f^{c,i} - \Delta_f^{d,i}) + e_z(b_f^j - b_f^{j-1}) + c_f^r \Delta_f^{d,i} = c_f^e(\alpha_f^i + \gamma_f^i) + e_z(b_f^j - b_f^{j-1}) + c_f^r \Delta_f^{d,i} \). The calculation of curfew penalty is as discussed previously, depending on the eventual time of arrival of the flight at its destination.
4. From (20), if $B_{i-1}^f = 1$, then $B_i^f = 1$ and $\beta_i^f = 0$. As before we observe that $f$ must be a member of $F_{z,A}$ for some $z$ and that $\Delta_{j,i}^{a_i} = \Delta_{j,i}^{a_i} = 0$. The incremental delay according to (20) is $\Delta_{j,i}^{g_{i+1}} = \Delta_{j,i}^{g_{i+1}}$. That is, the only option for this flight is to decide whether to increase the amount of time it spends grounded at its diversion airport. Hence the only terms that could have a positive value in the objective function for flight $f$ are $c_f^2 \Delta_{j,i}^{g_{i+1}}$ and $e_z(b_i^f - b_i^{f-1})$.

In this section, we have described MARFE and demonstrated its mathematical correctness. While it may appear to be a cumbersome and unwieldy integer programming model, the polyhedral structure bestowed on it by the choice of decision variables $(x_{f,i})$ means that in many cases (as observed computationally) it can be used to dynamically calculate new schedules in reasonable amounts of time. One such case study, with real schedules and real capacity data for Sydney airport in Australia, is discussed in the next section.

3. A case study: Sydney airport in Australia. We now illustrate the applicability of MARFE to optimising the recovery from schedule disruptions at Australia’s busiest airport, Sydney. Adverse weather is the main cause of capacity drops at Sydney.

Sydney airport characteristics. Prior to the events of September 2001, Sydney airport was the 38th busiest in the world. The number of daily movements was around 800. Sydney airport experienced a peak traffic level of more than 1000 daily movements during the Olympic Games held in September 2000. There is a government imposed restriction of 80 movements per hour. The movements are managed by slots; for arrivals as well as departures. Slot compliance is strictly enforced — if an airline misses a slot due to circumstances within its control at least 20% of the time, it loses that slot. The noise curfew applies between 11pm and 6am.

3.1. Experiments with the length of the warning period. The experiments were conducted on a 450 MHz machine running the RedHat Linux 5.2 operating system. The integer programming models were coded in the AMPL modelling language, and solved using the commercial optimisation package CPLEX (version 8). Each experiment (each run of MARFE) was completed in 15 minutes or less.

Flight Schedules for Sydney. We use actual flight schedules (arrival and departure information) at Sydney for a busy Monday (November 20, 2000). The flight data set and the flight timetables were obtained from reliable sources, including airline web pages and Sydney Airport Corporation Limited. Since air traffic at Sydney decreased sharply after September 2001, it was decided to use data from November 2000, because it was a period with heavy traffic. This data set consists of 791 movements at Sydney from 00:01 hours to 23:59 hours.

Costs of Delays. The detailed cost structure is given in Table A. The results reported here reflect value judgements embedded in the costs used, in particular the “common good perspective” that assigns higher costs to flights that carry more passengers. However, we stress that all costs are amenable to changes that reflect the given user’s priorities.
Table 4. Relative Aircraft Costs

<table>
<thead>
<tr>
<th>Aircraft Type</th>
<th>Ground Holding</th>
<th>Airborne Holding</th>
<th>Cancellation</th>
</tr>
</thead>
<tbody>
<tr>
<td>TurboProp</td>
<td>0.19</td>
<td>1.75</td>
<td>3.04</td>
</tr>
<tr>
<td>B737</td>
<td>0.50</td>
<td>1.75</td>
<td>8.00</td>
</tr>
<tr>
<td>B767</td>
<td>0.85</td>
<td>1.75</td>
<td>13.60</td>
</tr>
<tr>
<td>B747</td>
<td>1.35</td>
<td>1.75</td>
<td>21.60</td>
</tr>
</tbody>
</table>

For each flight type, the ground holding cost coefficient is the same for each time period, including the first. These coefficients range from 0.19 for flights involving small aircraft (such as turboprops), to 1.35 per time period for larger aircraft (such as Boeing 747s); they are directly proportional to aircraft passenger capacity. For each flight, the cost of cancellation is 16 times the cost of a single period of ground delay, which is equivalent to the cost of a ground delay of 4 hours.

The cost coefficient for airborne holding is the same for all flights. Flights approaching an airport are landed essentially in the order in which they arrive at the near-terminal airspace. Differential airborne holding cost coefficients (e.g., depending on aircraft size) could distort this precedence. The coefficient used here is 1.75 per 15 minute time period, which is greater than each of the ground holding cost coefficients and is also about three times the average ground holding cost. See Richetta and Odoni [11].

For the sake of simplicity of this illustration, diversions were not permitted as an intervention option.

Stage 1 of this experiment resulted in the cancellation of 13 flights and numerous ground holding delays, at a nominal cost of 109.375 units of our objective function. As was seen in Section 2.2, the objective function is a weighted aggregate of delay costs and its value is not directly related to monetary units.

Stage 2 Capacity Scenario Experiments. A number of different disruption scenarios were considered. The results of each of these have similar features, which we illustrate by means of the following discussion of one scenario. The scenario considered here is a sudden, short disruption that causes Sydney airport to be closed for an hour. That is, the capacity is equal to zero for each time period in the set \( \{t_1, \ldots, t_1 + 3\} \), with \( t_2 = t_1 + 4 \). The disruption commences at 8:00am (\( t_1 = 33 \)), which is a busy period at Sydney airport on a typical weekday. Note that since we consider only a single disruption, we suppress the superscript \( i \) on \( t_0 \), \( t_1 \) and \( t_2 \).

A series of experiments was conducted in which the parameters of the disruption (\( t_1 \) and \( t_2 \), and the capacity profile) are unchanged but the amount of warning (\( t_1 - t_0 \)) is varied. Note that although we use the term warning here, \( t_0 \) is the first time period in which action is taken to impose ground holding delays and cancellations, regardless of when information about the capacity emergency becomes available. Indeed, it is even possible that \( t_1 - t_0 < 0 \), that is, preemptive measures are commenced after the disruption has begun. Thus we include in our results some cases of “negative warning”. In particular, we portray the extreme (unrealistic) case of a large negative warning that corresponds to a complete lack of action being
taken to ameliorate airborne holding delays in response to the disruption. At the other extreme, we include the case of a large positive warning, meaning that the disruption to capacity is anticipated and reacted to sufficiently far in advance to completely prevent airborne holding.

The results of these experiments are best captured in Figure 2. In this figure, the relative cost breakdown — among airborne holding, cancellations and ground holding — is depicted against the variable that measures the length of the warning period as a multiple of the 15 minute intervals.

As expected, the total cost of airborne holding is proportionately high when there is little warning, decreasing gradually as the amount of warning increases. The total cost also decreases as the amount of warning and therefore the forward planning opportunities increase. As the airborne holding costs decrease a part of this reduction is transferred to a combination of ground holding and cancellation costs and the remainder constitutes a net saving. The extreme left bar in Figure 2 represents the cost of taking no preemptive recovery action to cope with the capacity restriction, resulting in all the costs accruing in the form of airborne holding costs. Recall that diversions were not enabled in these experiments.

**Figure 2.** Nominal stage 2 costs for one-hour closure at 8:00 am
Table 5. Visibility and Corresponding Arrival Capacity for Sydney

<table>
<thead>
<tr>
<th>Visibility Range (m)</th>
<th>0-500</th>
<th>500-1000</th>
<th>1000-4000</th>
<th>4000-7000</th>
<th>7000-10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrival Capacity (30 minutes)</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

An interesting and unanticipated result is that it seems that in most cases there is only a small benefit to be derived from having more than 6 time periods (90 minutes) of warning of the approaching disruption. Almost invariably, there is little change in the respective proportionate costs of ground holding, airborne holding and cancellation, as the number \( t_1 - t_0 \) of warning periods increases beyond this threshold, for quite a number of additional periods of warning. This can be observed from both the flattening of the bar graphs in Figure 2 and their breakdown for \( 6 \leq t_1 - t_0 \leq 12 \). This suggests that there is great value in obtaining accurate weather forecasts early enough to permit action to be taken 90 minutes before the onset of the disruption, but there is a diminishing return from longer range forecasts.

3.2. Experiments with actual schedule and weather. The experiments described in this subsection are the most realistic to date. In previous experiments, either the flight schedule or the weather data was synthetic, whereas in the experiments described here, both sets of data are actual.

The date of events chosen for testing is August 7, 1999 (Saturday) for Sydney airport. We used the published flight schedule for Sydney for this date, obtained from Airservices Australia, who also provided us with the actual flight arrival and departure information. The data were pre-processed to purge duplicate records. This resulted in 517 flights (movements) for the day consisting of 261 flight arrivals and 256 departures. The experimental setup was the same as that in the previous section. MARFE took less than five minutes to solve to optimality on the same computer.

Capacity Profile at Sydney on August 7, 1999: The archived weather data for this day were obtained from the Australian Bureau of Meteorology. Using visibility as the chief factor influencing the rate of arrivals, we computed the arrival capacity in a conservative manner. Table 5 exhibits the change in capacity profile with change in visibility. Data presented in Tables 5-6 were obtained from Airservices Australia. The actual arrival (or departure) capacities that we derived for the day are given in Table 6. Observe that the airport is effectively closed for five hours, from 3:30 am to 8:30 am, however, due to the curfew prior to 6:00 am, the closure is restrictive only for two and a half hours.

All parameters used in the reported experiments are listed in Table 7. We were unable to obtain reliable information on flight connections at Sydney.

As in the previous sub-section, the cost per unit period of delay is proportionally based on the seating capacity of aircraft. However, in the present experiments the cost of airborne holding is assumed to be twice the cost of ground holding, as suggested by an industry report published by Jenkins and Cotton in 2002 at Passur.com.
From the planned (scheduled) movements and the actual movements, we computed the delay experienced by each of the 517 flights. Figure 3 displays a comparison of delays that actually resulted with the delays that would have occurred if an optimal schedule from MARFE (generated at 5:00 am) were strictly implemented. It is clear that the latter produces significantly reduced delays. More details of this comparison can be found in Table 8. Note the reduction in the mean delay from 53 minutes (actual) to approximately 26 minutes for the optimized schedule and the reduction in the number of delayed flights from 416 to 128.

**Optimisation.** The optimisation models used here were:

- Stage 1 of MARFE, run in advance of the day of operations, using the expected capacity profile for the day. The result is a prescription of ground delays and cancellations, and the imposition of curfew penalties as needed. One should note that these penalties are chiefly to dissuade airlines from breaking the curfew, and are rarely imposed in reality.
Stage 2 of MARFE. The execution of this model is assumed to be complete at 5:00 am, one hour before the curfew in Sydney is lifted, on the day of the event, August 7, 1999. The capacities are zero between 3:30 and 8:30 am (see Table 8). Thus the period of adverse weather is in progress when the execution of MARFE is complete. We demonstrate that even if the optimisation model is used at such a late stage, enhanced recovery will still ensue. There are advantages to running MARFE at 5:00 am, even though the curfew is lifted only at 6:00 am. Flights that depart after 5:00 am and arrive at Sydney after 6:00 am can be groundheld — such flights would otherwise be held in airborne patterns if MARFE were to run at 6:00 am.

Table 8 compares the results from the two scenarios: (i) Actual, and (ii) Late optimisation with MARFE (during the period of adverse weather). The number of cancellations for the actual movements is an estimate, obtained in comparison to the number of movements at Sydney on Saturdays in the same season when the weather was perfect.

The cancellations in the optimized and unoptimized (actual) cases are of the same order of magnitude. However, the optimisation model yields much better results than the actual occurrence for all other performance indicators. In particular, optimisation achieves a reduction of (i) 70% in the number of flights with positive delay, (ii) 52% in the mean delay, and (iii) 34% in the total cost. Such results provide compelling reasons for air traffic service providers to consider optimisation techniques in daily flight scheduling and disruption recovery.
4. Conclusions. In this paper, we introduce a new Model for Adaptive Rescheduling of Flights in Emergencies (MARFE). The model enables us to optimally adapt to reduced capacity levels, almost as soon as a warning is issued of an impending capacity emergency. The latter covers any event where the previously forecasted airport capacity is reduced irrespective of whether this reduction is due to weather conditions, breakdown on a runway, or some other disruptive event.

MARFE has been run for an airport such as Sydney using actual daily schedules and typical capacity scenarios as input. Results indicate that the effect of an impending capacity emergency depends strongly on the amount of warning that has been given. Not surprisingly, short warning results in high airborne holding costs, whereas longer warnings enable us to significantly reduce the latter, albeit at the expense of somewhat increasing the ground holding cost. The benefits of additional warning dissipate for warning times of more than ninety minutes.

Collectively, the results described in this paper indicate that the proposed optimisation methodology has reached a stage where realistic scenarios of capacity emergencies can be solved in real time for an airport such as Sydney. Furthermore, we have demonstrated that implementations of the solutions of these models have the potential to offer significant benefits to the flying public.

In principle, MARFE could be enhanced by modelling the following additional features:

- Allowing flights to arrive earlier than their scheduled arrival times, before an impending capacity emergency.
- Considering the cost of delays to be generalized (convex costs, for example), as opposed to simple piecewise linear costs.
- Treating departure capacities separately from arrival capacities.
- Identifying cycles (beginning and ending at the same airport) of successor flights that could be cancelled without cancelling subsequent successor flights.

Another issue to be considered is that of implementing “decision equity” between the airlines. Flights belonging to a single airline (or a group of airlines) should not suffer excessive amounts of delays or an unfairly large number of cancellations consistently. For example, the proportion of flights cancelled should approximately be the same for all airlines over a certain period (say, a week). This could be
implemented by using a randomised approach to choose flights to be delayed, for instance.

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