Estimation of Induction Motor Parameters Using Hybrid Algorithms for Power System Dynamic Studies

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***Abstract*— This paper proposes a hybrid Newton-Raphson and genetic algorithm for the estimation of double cage induction motor parameters from commonly available manufacturer data. The hybrid algorithm was tested on a large data set of 6,380 IEC and NEMA motors and then compared with a baseline Newton-Raphson algorithm. The simulation results show that while the proposed hybrid algorithm is more computationally intensive, it does make significant improvements to convergence and error rates.**

Keywords*— *Induction motor; parameter estimation; hybrid algorithm

I. INTRODUCTION

The equivalent circuit parameters of induction motors are often required in dynamic power system studies, particularly for motor starting simulations. However typically, the only motor data available to the user is from brochures or manufacturer data sheets and rarely are the full equivalent circuit parameters supplied. Furthermore, most simulation software packages employ a motor model with constant parameters applicable over the full range of slip values. But in reality, the equivalent circuit parameters of a motor are not constant and depend on a number of factors, e.g. slip, temperature, saturation, etc [1] [2]. Therefore, in order to use simulation software, it is necessary to estimate constant parameters such that the calculated performance characteristics (e.g. breakdown torque, locked rotor current, etc) fairly closely matches the data supplied by the manufacturer.

Over the years, a number of motor parameter estimation algorithms have been proposed (for example, [6], [1], [2], [7] and [4]). The de facto standard that has emerged from the literature, and which has been adopted by the majority of commercial software packages, is to use an algorithm based on the Newton-Raphson approach.

In this paper, the standard Newton-Raphson approach is extended by combining it with a genetic algorithm to estimate two parameters that the Newton-Raphson algorithm by itself cannot. The new hybrid algorithm is then tested on a large data set of 6,380 motors taken from the EURODEEM and MotorMaster databases [10]. The results are compared with a baseline Newton-Raphson algorithm in terms of convergence and average squared errors.

II. DOUBLE CAGE INDUCTION MOTOR MODEL

It has been previously shown that the single cage model is insufficient to capture both the starting and breakdown torque characteristics of a squirrel-cage induction motor without introducing significant errors [3]. The double cage model is therefore more appropriate to represent the full torque-speed characteristics of a motor with a single set of constant parameters.

The steady-state, double cage equivalent circuit model shown in Figure 1 is used in this paper with eight slip-invariant parameters valid over the full range of slip values (i.e. from 0 to 1 pu).

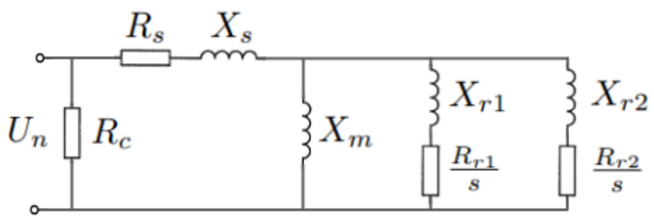

Fig. 1. Double cage motor equivalent circuit (eight parameter model)

In the equivalent circuit, the inner cage leakage reactance X_{r1} is always higher than the outer cage leakage reactance X_{r2} but the outer cage impedance is typically higher than the inner cage impedance on starting. These conditions can be resolved by including the following two inequality constraints in the model [4]:

$$X_{r1} > X_{r2}$$

$$R_{r2} > R_{r1}$$

In order to estimate motor efficiency, the core (and mechanical) losses also need to be included in the model. For simplicity, the core (and mechanical) losses are represented as a single shunt resistance R_c at the input of the equivalent circuit [5].

III. PARAMETER ESTIMATION PROBLEM FORMULATION

The characteristics of an induction motor are normally provided by manufacturers in the form of a standard set of performance parameters, with the following being amongst the most common:

- Nominal voltage, U_n (V)
- Nominal frequency, f (Hz)
- Rated asynchronous speed, n_{fl} (rpm)
- Rated (stator) current, $I_{s,fl}$ (A)
- Rated mechanical power, $P_{m,fl}$ (kW)
- Rated torque, T_n (Nm)
- Full load power factor, $\cos \phi_{fl}$ (pu)
- Full load efficiency, η_{fl} (pu)
- Breakdown torque, T_b / T_n (normalised)
- Locked rotor torque, T_{lr} / T_n (normalised)
- Locked rotor current, I_{lr} / $I_{s,fl}$ (pu)

While all of the parameters in this set can be used in the estimation procedure, there are only six independent magnitudes that can be formed from them: $P_{m,fl}$, Q_{fl}, T_b, T_{lr}, I_{lr} and η_{fl} [4]. Refer to Appendix 1 for details on the relationships between the parameters and how these six independent magnitudes are calculated from the equivalent circuit model.

The six independent magnitudes can be used to formulate the parameter estimation in terms of a non-linear least squares problem, with a set of non-linear equations of the form $\boldsymbol{F}(\boldsymbol{x}) = \boldsymbol{0}$:

$$f_1(\boldsymbol{x}) = P_{m,fl} - P(s_f) = 0 \quad (1)$$

$$f_2(\boldsymbol{x}) = Q_{fl} - Q(s_f) = 0 \quad (2)$$

$$f_3(\boldsymbol{x}) = T_b - T(s_{max}) = 0 \quad (3)$$

$$f_4(\boldsymbol{x}) = T_{lr} - T(s = 1) = 0 \quad (4)$$

$$f_5(\boldsymbol{x}) = I_{lr} - I(s = 1) = 0 \quad (5)$$

$$f_6(\boldsymbol{x}) = \eta_{fl} - \eta(s_f) = 0 \quad (6)$$

Where $\boldsymbol{F}(\boldsymbol{x}) = (f_1, f_2, f_3, f_4, f_5, f_6)$ and
$\boldsymbol{x} = (X_s, X_m, R_{r1}, X_{r1}, R_{r2}, R_c)$

In the formulation above, we have six independent equations, but eight unknown parameters (since R_s and X_{r2} are not part of the solution vector $\boldsymbol{x}$).

IV. BASELINE NEWTON-RAPHSON ALGORITHM

The Newton-Raphson (NR) algorithm proposed by Pedra in [5] is used as the baseline algorithm in this paper. This algorithm was selected because of its completeness, numerical accuracy and robustness compared to previously proposed methods (for example, in [6], [1] and [2]). Furthermore, the algorithm can be applied using commonly available manufacturer data, whereas other algorithms require more detailed data that may not be readily available.

The NR algorithm is an iterative method where each iteration is calculated as follows:

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k - h_n \boldsymbol{J}^{-1} \boldsymbol{F}(\boldsymbol{x}^k) \quad (7)$$

Where $\boldsymbol{x}^{k+1}$ is the solution at the $(k+1)$th iteration
$\boldsymbol{x}^k$ is the solution at the k-th iteration
h_n is the step-size coefficient (more on this later)
$\boldsymbol{J}$ is the Jacobian matrix evaluated with $\boldsymbol{x}^k$

The Jacobian matrix $\boldsymbol{J}$ has the general form:

$$\boldsymbol{J} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_6} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_6}{\partial x_1} & \cdots & \frac{\partial f_6}{\partial x_6} \end{bmatrix}$$

For systems where it is impractical to compute the exact partial derivatives analytically, a numerical approximation may be used with finite difference equations:

$$\frac{\partial f_i}{\partial x_j} \approx \frac{f_i(\boldsymbol{x}+\boldsymbol{\delta}_j h) - f_i(\boldsymbol{x})}{h} \quad (8)$$

Where $\boldsymbol{\delta}_j$ is a vector of zeros with a single non-zero value of 1 at the j-th element
h is a constant of very small absolute value (i.e. 1×10^{-5} is used in this paper.

A. Linear Restrictions

It was noted earlier that there are eight unknown parameters but only six independent equations. Therefore, the non-linear system is underdetermined. In order to make the system solvable, the values of R_s and X_{r2} need to either be fixed (i.e. assumed to be known) or calculated from relationships with other parameters.

In the baseline algorithm, Pedra proposes imposing the following linear restrictions [4]:

$$R_s = k_r R_{r1} \qquad (9)$$

$$X_{r2} = k_x X_s \qquad (10)$$

Where $k_r = 0.5$ and $k_x = 1$ are the constants suggested in [5].

B. Parameter Constraints

The inequality constraints of the double cage model ($X_{r1} > X_{r2}$ and $R_{r2} > R_{r1}$) can be implicitly included into the formulation by a simple change of variables [4]:

$$x_1 = R_{r1}$$

$$x_2 = R_{r2} - R_{r1}$$

$$x_3 = X_m$$

$$x_4 = X_s$$

$$x_5 = X_{r1} - k_x X_s$$

$$x_6 = R_c$$

Furthermore, only the absolute values of the parameter estimates are used to ensure that no negative parameters are estimated.

C. Initial Conditions

The initial parameter estimates are selected as follows based on [4]:

$$R_{r1} = \frac{U_n s_f}{P_{m,fl}}$$

$$X_m = \frac{U_n}{Q_{fl}}$$

$$X_s = 0.05 X_m$$

$$R_s = k_r R_{r1}$$

$$R_{r2} = 5 R_{r1}$$

$$X_{r1} = 1.2 X_s$$

$$X_{r2} = k_x X_s$$

$$R_c = 10$$

D. Limitations of the Newton-Raphson Algorithm

In the NR algorithm, linear restrictions are imposed on R_s and X_{r2} in order to make the underdetermined system of equations solvable. It was shown in [8] that the double cage model with core losses has 8 minimum independent variables (MIVs). Therefore, by constraining R_s and X_{r2} with linear restrictions, we are also constraining the solution space by two degrees of freedom. Thus without the linear restrictions, a solution could potentially exist to an otherwise non-converging problem.

V. PROPOSED HYBRID ALGORITHM

The proposed hybrid algorithm attempts to overcome the limitations of the NR algorithm by applying an evolutionary method (i.e. a genetic algorithm) to select R_s and X_{r2} In other words, the baseline Newton-Raphson algorithm is run with fixed values for R_s and X_{r2} which are in turn iteratively selected using a genetic algorithm in an outer loop. A flowchart of the proposed hybrid algorithm is shown in Figure 2. A more detailed description of the proposed algorithm follows.

Genetic algorithms can be binary coded where the solution paramaters are quantized into binary strings (for example, in [9]). However, the equivalent circuit parameters in a motor are continuous parameters and not naturally quantized. Thus, binary coding necessarily imposes limits on the precision of the parameters (i.e. due to the chosen length of the binary string). For this reason, a continuous parameter genetic algorithm is used instead.

An initial population of n_{pop}estimates for R_s and X_{r2} are randomly sampled from a uniform distribution with upper and lower limits as shown in Table I. Each pair of estimates is referred to as a member of the population.

TABLE I. RANGE OF INITIAL PARAMETER ESTIMATES

Parameter	Range of Initial Estimate	
	Lower Bound	***Upper Bound***
R_s	0	0.15
X_{r2}	0	0.15

The NR algorithm is then run on each member of the population. The fitness of each member (in terms of the squared error $\boldsymbol{F'F}$) is calculated and ranked. The lowest fitness members are discarded and the rest are retained to form the mating pool for the next generation (there are n_{pool} members in the mating pool).

The fittest n_e members in the mating pool are retained for the next generation as **elite children**. Of the remaining $n_{pop} - n_e$ children to be created for the next generation, c_f% will be produced by crossover and the rest $(1 - c_f)$% by mutation. The proportion c_f is called the **crossover fraction**.

1) ***Crossover****:* in the crossover process, two members of the mating pool are randomly selected and combined by taking a random blend of each member's parameters, e.g. the crossover of parameter R_s:

$$R_{s,child} = \alpha R_{s,parent1} + (1 - \alpha) R_{s,parent2} \quad (11)$$

Where α is a random variable selected from a uniform distribution over the interval [0,1].

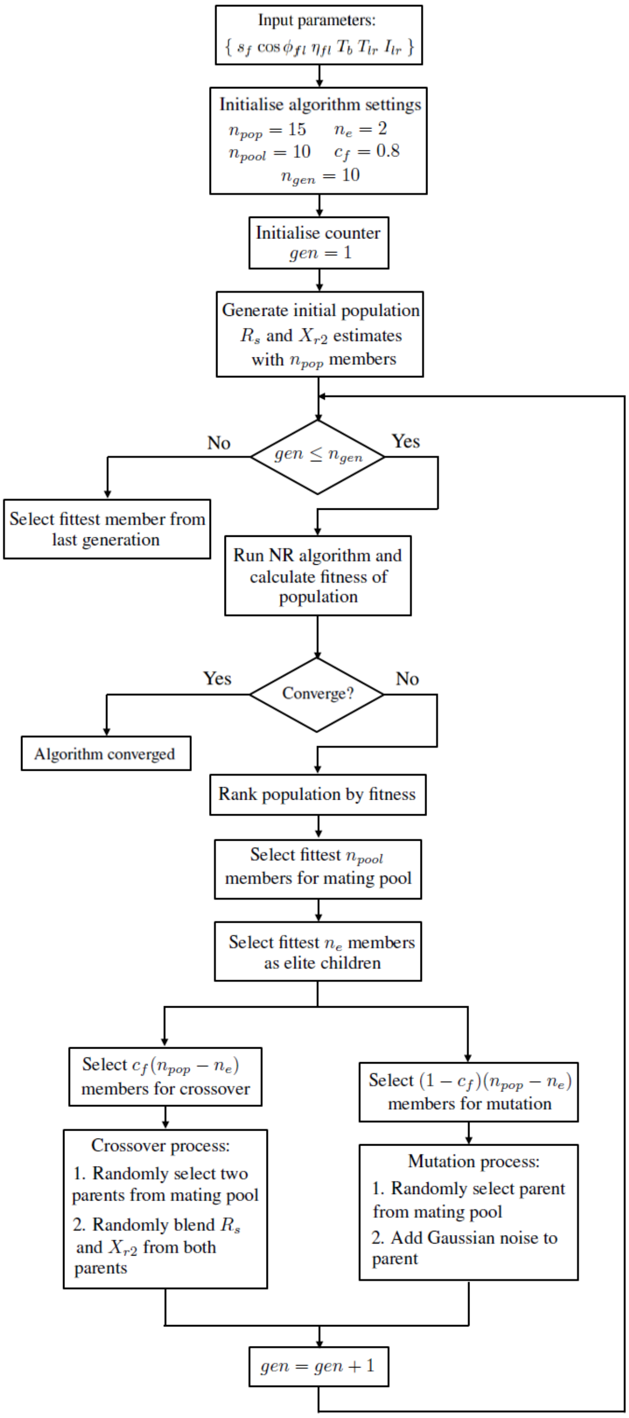

Fig. 2. Flowchart for hybrid algorithm

2) ***Mutation****:* in the mutation process, a member of the mating pool is randomly selected and its parameters are mutated by adding Gaussian noise with standard deviations of 0.01.

The NR algorithm is then run for the next generation of estimates for R_s and X_{r2}. The fitness is calculated and the process repeats itself for n_{gen} generations. If at any point during the process the NR algorithm converges, then the hybrid algorithm stops and selects the parameter estimates from the converged NR algorithm as the solution. Otherwise, the parameter estimates yielding the best fitness after n_{gen} generations are selected.

The default settings for the hybrid algorithm implemented in this paper are shown in Table II.

TABLE II. DEFAULT SETTINGS FOR HYBRID ALGORITHM

Setting	Setting Description	Default Value
n_{pop}	Population of each generation	15
n_{pool}	Number of members in the mating pool	10
n_e	Number of elite children	2
c_f	Crossover fraction	80%
n_{gen}	Maximum number of generations	10

VI. COMPUTER SIMULATION

The baseline NR and proposed hybrid algorithms were tested on a large data set from the EuroDEEM and MotorMaster databases (version 1.0.17 - 4 April 2007) with a mixture of IEC and NEMA type motors [10]. From the original set, the motor data was conditioned by eliminating duplicate records, removing motors without power factor, efficiency or torque data and removing motors with strange or inconsistent data (e.g. full load torque greater than breakdown torque, asynchronous speed greater than synchronous speed, etc). After data cleansing, the final data set consisted of motors with nominal ratings from 0.37kW to 1000kW, and the following total quantities:

- 4,002 IEC 50Hz motors
- 2,378 NEMA 60Hz motors

A. Simulation Results

Table III shows the results of the simulations performed on both the IEC and NEMA motor data set. The results present the rates of convergence and average squared errors of the proposed hybrid algorithm compared with the baseline NR algorithm.

It can be seen from Table III that the proposed hybrid algorithm significantly outperforms the baseline NR algorithm, both in terms of convergence rates and squared errors. In the IEC data set, the convergence rate is almost doubled when using the hybrid algorithm, while in the NEMA data set, there is a 54% improvement.

TABLE III. SIMULATION RESULTS FOR BASELINE NR AND HYBRID ALGORITHMS

Case	**IEC Motors**		**NEMA Motors**	
	Convergence	***Average Squared Error***	***Convergence***	***Average Squared Error***
Baseline NR algorithm, $k_r = 0.5$, $k_x = 1$	685 (17.1%)	0.5411	751 (31.6%)	0.2514
Hybrid Algorithm	1363 (34.1%)	0.0625	1159 (48.7%)	0.0282

The average squared errors in the hybrid algorithm are 8.7 and 8.9 times lower than the baseline NR algorithm for the IEC and NEMA data sets respectively.

There is a computational cost for the improvement in performance since the hybrid algorithm is considerably more computationally intensive than the baseline NR algorithm. This is because the evolutionary part of the hybrid algorithm must run the NR algorithm multiple times for each generation. For example, based on the default settings as shown in Table II the hybrid algorithm may have to perform up to $n_{pop} \times n_{gen} = 10 \times 15 = 150$ NR algorithms. This would occur in the worst case condition when the hybrid algorithm fails to converge. Nevertheless, as motor parameter estimation for the purpose of system studies is not a particularly time-critical task, then perhaps algorithm performance is a much larger driver than computational burden and run time.

In this paper, the default criteria for convergence (i.e. error tolerance) is a squared error value of 1×10^{-5}. When the criterion for convergence is relaxed, one would expect a corresponding increase in the convergence rate and this is in fact what is observed in the results. Figures 3 and 4 show the convergence rate as a function of the error tolerance for IEC and NEMA motors respectively. As expected, convergence rates increase as the error tolerance is also increased. The slope of the curve is relatively flat until the error tolerance is increased to 1×10^{-3}. Note also that the hybrid algorithm outperforms the baseline NR algorithm in all cases.

VII. CONCLUSION

In this paper, a new hybrid algorithm for motor parameter estimation was proposed and compared against a baseline Newton-Raphson algorithm. Both algorithms were tested on a data set comprising 6,380 IEC and NEMA motors.

When the baseline Newton-Raphson parameter estimation algorithm was applied to the data set, it was found that the algorithm had poor convergence properties (17.1% of IEC motors and 31.6% of NEMA motors converged). But by adopting the proposed hybrid algorithm, significant improvements were made to both the algorithm's convergence (34.1% of IEC motors and 47.1% of NEMA motors) and average squared error. However, the drawback with the hybrid algorithm is that it is considerably more computationally taxing.

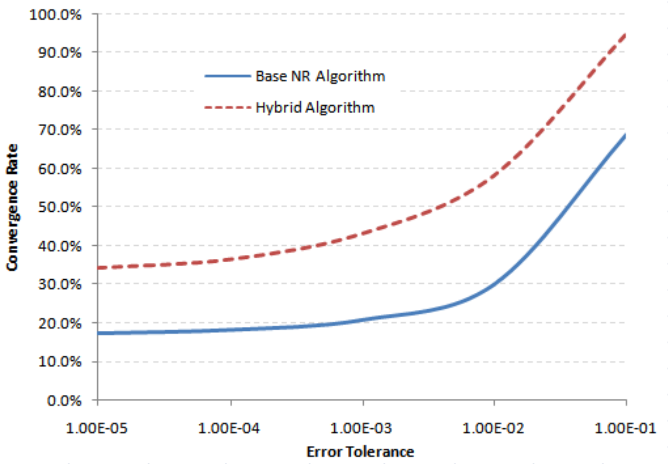

Fig. 3. Convergence rate versus error tolerance plot for IEC motors

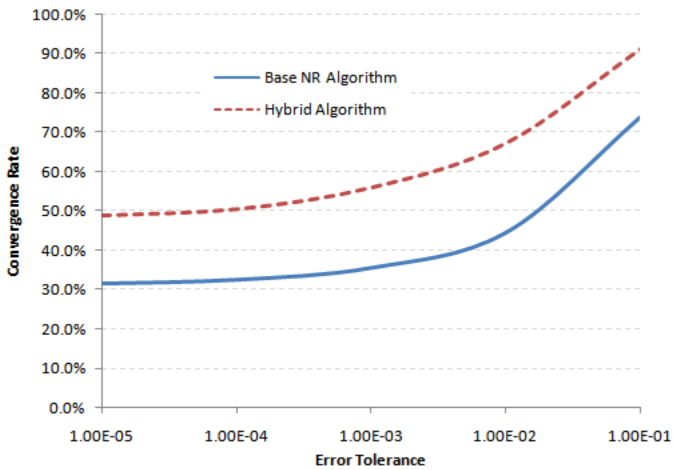

Fig. 4. Convergence rate versus error tolerance plot for NEMA motors

APPENDIX 1: DOUBLE CAGE MODEL EQUATIONS

Stator and rotor currents at slip s can be readily calculated from the equivalent circuit (for example, see [6]). The per-unit torque can be calculated from the rotor currents as follows:

$$T(s) = \frac{R_{r1}}{s} I_{r1}^2 + \frac{R_{r2}}{s} I_{r2}^2$$

Quantities for per-unit active power, reactive power and power factor can be calculated as follows:

$$S(s) = U_n I_s(s)^*$$

$$P(s) = T(s)(1 - s)$$

$$Q(s) = \Im\mathfrak{m}\{S(s)\}$$

$$\cos\phi(s) = \frac{\Re\mathfrak{e}\{S(s)\}}{\|S(s)\|}$$

Synchronous speed and per-unit slip is calculated as follows:

$$n_s = \frac{120f}{p}$$

$$s_f = 1 - \frac{n_{fl}}{n_s}$$

Calculating the slip at maximum torque s_{max} is found by solving the equation:

$$\frac{dT}{ds} = 0$$

And under the condition that the second derivative $\frac{d^2T}{ds^2} < 0$. In the double cage model, the solution to this equation is not trivial and it is more convenient to use an estimate, e.g. based on an interval search between $s = 0$ and $s = 0.5$.

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