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Phung, T. H. and I. A. Sultan (2018). "Characterization of limaçon gas expanders with consideration to the dynamics of apex seals and inlet control valve." *Journal of Engineering for Gas Turbines and Power* 140(12).

Which has been published in final form at: <https://doi.org/10.1115/1.4040418>

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Characterisation of Limaçon gas expanders with consideration to the dynamics of apex seals and inlet control valve

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Abstract

Limaçon machine, of which the relative motion between the rotor and housing follows the limaçon curve, belongs to a class of rotary positive displacement machines. The profiles of rotors and housings of those machines can be constructed of either limaçon or circular curves, hence the names: limaçon-to-limaçon, circolimaçon, and limaçon-to-circular machines. This paper presents the investigation into the thermodynamic performance of the limaçon-to-circular machines with the presence of apex seals and inlet valve.

This paper sets out by briefly introducing the limaçon technology and the construction of the limaçon-to-circular machine working volume. The mathematical descriptions of ports' positions and areas have also been introduced. The paper then discusses the flow and phase composition of working fluid through the working chambers as well as how the fluid velocity is modelled and calculated. Then the seal dynamic model and response of inlet valve are presented followed by the machine thermodynamic model. A case study has also been presented to show the responses of seals and inlet valve during the machine operation.

Keywords: Rotary machine; limaçon-to-circular; limaçon machine; limaçon motion; pump; gas expander; compressor; positive displacement; optimisation; thermodynamics.

Article type: Research paper

1. Introduction

Over the past years, the consumption of fossil fuel continues to grow (International Energy Agency, 2015; Department of Industry and Science, 2015) due to the increase in energy demand in both heat and electricity forms.

The growth in non-renewable resource consumption together with the low efficiency of the current energy conversion

processes result in enormous amount of excess heat is wasted and more greenhouse gas makes its way to the atmosphere. The energy generating activities are definitely key factors that drive the changes in our climate system and escalating risk for different aspect of life (IPCC, 2015; Climate Council, 2015).

From both the environmental and the technological point of view, it is essential to reduce the consumption of non-renewable resources by increasing the efficiency of energy conversion processes as a short-term strategy. The process efficiency can be increased by mean of heat recuperation – a practice that captures the previously overlooked waste heat to produce extra mechanical or electrical power. Organic Rankine Cycle (ORC) based micro power plants that utilise organic fluids to transfer heat and expand in the expander to give off work are often employed (Mathias, Johnston, Cao, Priedeman, & Christensen, 2009). Due to the relatively low temperature of low-grade heat, components of ORC micro power plants are often exposed to multi-phase flow; such a condition allows fluid droplets to be formed, which is not suitable for turbine technology. Lemort et al. (2009) and Sultan and Schaller (2011; 2012) in their published work have pointed out that the perfect candidates for low working fluid temperature, multi-phase flow, and low flow rate conditions are positive displacement machines, which include different embodiments of limaçon machines.

This paper investigates the output power and the efficiency of the limaçon-to-circular machine. A mathematical model has been developed based on the thermodynamic performance, seal dynamic, and inlet valve dynamic of such a machine. A case studies has been presented to test the reliability of such a model.

2. The limaçon-to-circular machine's working volume

Detailed in Figure 1 is a limaçon-to-circular machine and its geometrical aspects. As shown in the figure, the limaçon chord, $p_l p_t$, whose length is $2L$ is allowed to rotate and slide about the limaçon pole, o , which is the origin of a stationary frame XY . The profile of the limaçon housing is formed by the trace of the apices, p_l or p_t . Additionally, another frame namely $X_r Y_r$ is firmly attached to the rotor chord at its centre point, m , and moves with the chord. The angle θ measured from X -axis to X_r -axis followed the right-hand sense is the angular displacement of the machine rotor. When the rotor is in motion, its centre point, m , stays kinematically attached to the base circle of radius, r . It is worth noticing that, the two lobes of such a limaçon-to-circular machine is constructed of two circular segments of radius, R , which take the chord, $p_l p_t$, as the mirror line; hence, the machine received the name limaçon-to-circular machine. Phung, Sultan, and Boretti (2016) have pointed out that the housing and rotor of such

a machine have distinct profiles (i.e.: limaçon housing, circular segment rotor) and the rotation of the rotor inside the

housing follows the limaçon housing; therefore, during operations this machine may encounter rotor-housing interference. As such, Phung et al. (2016) have introduced a clearance value, C , to their calculation to yield the value of R as follows:

$$R = (1 - C_L) \sqrt{L^2 + \frac{4r^2 a^2}{1 - 4b^2}} \quad [1]$$

where $0 < b = \frac{r}{L} = 0.25$ is the limaçon aspect ratio, $a > 1$, and $C_L = \frac{C}{L}$. Consequently, the volume of the working chamber below the rotor chord, V_{net} , at any rotor angle, θ , can be expressed as follows:

$$V_{net_b} = HL^2 \left\{ \left[\left(b^2 + \frac{1}{2} \right) \pi - 4b \cos \theta \right] - (1 - C_L) \left[\tan^{-1} \left(\frac{\sqrt{1 - 4b^2}}{2ab} \left(1 + \frac{4a^2 b^2}{1 - 4b^2} \right) \right) - \frac{2ab}{\sqrt{1 - 4b^2}} \right] \right\} \quad [2]$$

where H is the expander's housing depth measured perpendicular to the page.

Figure 1: A limaçon-to-circular expander

Section 3 below introduces a model to calculate the instantaneous port area of the limaçon-to-circular machine as seen from the working chamber.

3. Inlet and outlet ports' cross-sectional areas calculation

Sultan and Schaller (2011) have developed a concept to calculate the cross-sectional areas of the inlet and outlet ports of the limaçon-to-limaçon machine. The proposed concept treats both ports without discrimination; hence, no specific reference has been made to either port. As such, this paper will employ the similar approach to calculate the port areas for the limaçon-to-circular machine.

The radial positions of the port leading and trailing edges are defined by the two position vectors, $\mathbf{R}_l$ and $\mathbf{R}_t$, which are described in Figure 2. The leading edge is defined as the one that the rotor chord leading apex, p_l , meets first during one cycle of operation. Those port edges vectors $\mathbf{R}_l$ and $\mathbf{R}_t$, can be expressed as follows:

$$\begin{cases} \mathbf{R}_l = L(2b \sin \theta_l + 1)\hat{\mathbf{R}}_l \\ \mathbf{R}_t = L(2b \sin \theta_t + 1)\hat{\mathbf{R}}_t \end{cases} \quad [3]$$

where θ_l is the leading edge, θ_t is the trailing edge, $\hat{\mathbf{R}}_l = \begin{bmatrix} \cos \theta_l \\ \sin \theta_l \\ 0 \end{bmatrix}$ and $\hat{\mathbf{R}}_t = \begin{bmatrix} \cos \theta_t \\ \sin \theta_t \\ 0 \end{bmatrix}$ are the unit vectors.

The angular location of the leading edge, $\theta_l < \frac{\pi}{2}$, is defined first, followed by the port angular width, $\Delta\theta_p$. Hence, the angular position of the trailing edges can be calculated as: $\theta_t = \theta_l + \Delta\theta_p$. The port length, L_p , is given within the condition bounded by the rotor depth, H , in which $L_p < H$. The port width, W , is calculated from the two edges vectors, $\mathbf{R}_l$ and $\mathbf{R}_t$, as:

$$W = |\mathbf{R}_l - \mathbf{R}_t| \quad [4]$$

In this paper, the authors have utilised the semicircular shape for the port ends as described in Figure 2; hence, the full cross-sectional area of the port, A_f , can be calculated as follows:

$$A_f = L_p W - W^2 \left(1 - \frac{\pi}{4}\right) \quad [5]$$

In order to study the rotor-port interaction, which is the relation between the leading apex of the rotor and the port edges, the position of the rotor leading and trailing edges are defined as follows:

$$\begin{cases} \mathbf{P}_l = L(2b \sin \theta + 1)\hat{\mathbf{X}}_r \\ \mathbf{P}_t = L(2b \sin \theta - 1)\hat{\mathbf{X}}_r \end{cases}$$

where $\hat{\mathbf{X}}_r = \begin{bmatrix} \cos \theta \\ \sin \theta \\ 0 \end{bmatrix}$ is a unit vector along the rotor chord, which is shown in Figure 2. The relative locations of the rotor leading apex with respect to the port edges can now be defined using the two scalar quantity, s_l and s_t , as follows:

$$\begin{cases} s_l = (\hat{\mathbf{R}}_l \times \hat{\mathbf{X}}_r) \cdot (\hat{\mathbf{X}}_r \times \hat{\mathbf{Y}}_r) \\ s_t = (\hat{\mathbf{R}}_t \times \hat{\mathbf{X}}_r) \cdot (\hat{\mathbf{X}}_r \times \hat{\mathbf{Y}}_r) \end{cases}$$

[7]

where $\hat{\mathbf{Y}}_r = \begin{bmatrix} -\sin \theta \\ \cos \theta \\ 0 \end{bmatrix}$ is a unit vector along the axis perpendicular to the rotor chord (Figure 2).

The port areas can now be determined from one of the following four cases:

- Case 1: when $s_l \geq 0$ and $s_t \geq 0$, it is suggested that the port is fully open to the control volume. In this case the port instantaneous area, A_p , can be set equal to the full area, A_f .
- Case 2: when $s_l \geq 0$ and $s_t < 0$, it is suggested that the port is being progressively opened to the control volume. In this case the port area, A_p , can be approximated using the equation $A_p = A_f \frac{|P_l - R_l|}{W}$.
- Case 3: when $s_l < 0$ and $s_t \geq 0$, it is suggested that the port is being progressively shut off from the control volume. In this case the port area, A_p , can be approximated using the equation $A_p = A_f \frac{|P_t - R_t|}{W}$.
- Case 4: when $s_l < 0$ and $s_t < 0$, it is suggested that the port is totally shut off from the control volume. The port area, A_p , in this case is equal to zero (i.e.: $A_p = 0$).

Figure 2: The limaçon-to-circular port area

The above four cases of the port areas can be summarised as follows:

$$\left\{ \begin{array}{l} \text{if } s_l \geq 0 \text{ and } \left\{ \begin{array}{l} s_t \geq 0 \text{ then } A_p = A_f \\ s_t < 0 \text{ then } A_p = A_f \frac{|P_l - R_l|}{W} \end{array} \right. \\ \text{if } s_l < 0 \text{ and } \left\{ \begin{array}{l} s_t \geq 0 \text{ then } A_p = A_f \frac{|P_t - R_t|}{W} \\ s_t < 0 \text{ then } A_p = 0 \end{array} \right. \end{array} \right.$$

[8]

The velocity of fluid flow through the port will be studied in section 4 below.

4. Fluid velocity through ports

The flow of the working fluid through the ports of the limaçon machine encounters a sudden expansion and a change in direction, this type of flow is similar to that of the globe valves; therefore, these limaçon-to-circular machines' ports can be approximated as globe valves rather than orifices. McNeil (1999), who also referred to the work of Fairhurst (1983) on globe valves, has published work on the approximation of flow in orifice plates, gate valves as well as globe valves.

The flow through ports undergoes an energy conversion process, which can be described as an equation to calculate the velocity at the downstream side of the ports:

$$U = \sqrt{2(h_u - h_d)} = \sqrt{2\Delta h}$$

[9]

where h_u and h_d are the up-stream and down-stream isentropic enthalpy, respectively

Δh is an isentropic enthalpy drop along the flow direction

When the losses due to friction and the change in cross-sectional area are taken into account, equation [9] can be re-written as a function of the enthalpy drop and the loss coefficient, $K_p = N_p f$, as follows (Massoud, 2005):

$$U = \sqrt{\frac{2\Delta h}{K_p}} \quad [10]$$

where N_p is a specific number given for a specific type of flow and f is a friction factor for turbulent flow. The friction factor, f , can be calculated using the Colebrook correlation (Munson, Rothmayer, Okiishi, & Huebsch, 2013) as follows:

$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}} \right) \quad [11]$$

where ε is the surface roughness, D is the diameter or hydraulic diameter of the pipe or the flow container, and Re is the Reynolds number. Equation [11], however, requires iteration to solve for the friction factor, f . Hence, Massoud (2005) suggested that in most engineering applications where smooth pipes are utilised and the flow is fully developed turbulent, the friction factor can be expressed using the McAdams formulation:

$$f = \frac{0.184}{Re^{0.2}} = \frac{0.184}{\left(\frac{\rho U D}{\mu}\right)^{0.2}} \quad [12]$$

From the definition of Reynolds number, $Re = \frac{\rho U D}{\mu}$, equations [10] and [12] can be utilised to find the velocity of flow through the inlet port of the limaçon-to-circular machine as:

$$U_i = \begin{cases} \left(\frac{2\Delta h}{0.184 N_p} \right)^{\frac{5}{9}} \left(\frac{\rho_{id} D_{id}}{\mu_{id}} \right)^{\frac{1}{9}} & \text{if } \sqrt{2\Delta h} < U_s \\ \frac{U_s^{\frac{11}{10}}}{\sqrt{0.184 N_p}} \left(\frac{\rho_{id} D_{id}}{\mu_{id}} \right)^{\frac{1}{10}} & \text{if } \sqrt{2\Delta h} > U_s \end{cases} \quad [13]$$

where subscript i represents the flow through the inlet port.

ρ_{id} , D_{id} , and μ_{id} are the down-stream density, hydraulic diameter, and viscosity, respectively.

U_s is the speed of sound on the down-stream side of the port

The above equation can also be used to calculate the velocity of flow through the outlet port by replacing the subscript i for the inlet, by o for the outlet. Equation [13] can be utilised for both single and two-phase flow situations. In two-phase flow condition, the homogeneous flow model (Fairhurst, 1983) is to be employed and used in line with equations [12] and [13], in which the equivalent viscosity is obtained using the McAdams' equation as follows:

$$\frac{1}{\mu_{id}} = \frac{x}{\mu_{vap}} + \frac{1-x}{\mu_{lid}} \quad [14]$$

where μ_{id} is the equivalent down-stream viscosity of the inlet port

μ_{vap} , and μ_{lid} are the viscosities of the vapour and liquid phase, respectively

$$x = \frac{\text{mass flowrate of the gas phase}}{\text{total mass flowrate}} = \frac{\dot{m}_{gas}}{\dot{m}_{liquid} + \dot{m}_{gas}} \text{ is the dryness fraction (Ryley, 1964)}$$

Additionally, the speed of sound of the two-phase flow can be calculated either by using the available REFPROP functions offered by Lemmon et al. (2010) or based on the equilibrium of mechanical, thermal, and fluid phase assumptions, such an approach is detailed in the published work by Lund and Flatten (2010). The accuracy of the speed of sound can also be improved by using the two methods simultaneously.

With the utilisation of the loss coefficient, K_p , and the McAdams formulation, the velocity of fluid flows through the inlet and outlet ports can be calculated using equation [13] in a non-iterative manner. Moreover, the employment of the enthalpy difference instead of pressure difference has excluded the need of calculating the expansion factor during the expansion process in two-phase flow condition. Hence, in the following section, the flow velocity equation [13] will be put in to use in accordance with the continuity and energy equations in order to obtain a thermodynamic model for the limaçon-to-circular machine.

5. The seal dynamic model

In order to reduce the leakages and improve the efficiency, the limaçon machine will require sealing for both the apex-housing and the side gaps. The side seals experience relatively constant pressure difference and no gap

changes during machine operation. On the other hand, the apex-seals are subject to severe dynamic effects as they undergo alternating pressures from the machine chambers as well as changing distance from the seal apex to its corresponding point on the housing. This gives rise to the need to investigate the impact of the motion of the seal relative to the rotor, housing, and seal-groove on the machine performance and functioning. Even though the dynamic model of the apex seal is fairly complex and is a topic of a separate publication, an attempt is made here to summarise the seal dynamic model in such a manner which enables the reader to capture the fundamental aspects of this model. The seal axial length d_s can also be described as $d_s = d_{s_1} + d_{s_2}$ where d_{s_1} and d_{s_2} are measured from the seal centre of gravity (CG) to either end of the seal (Figure 4). Figure 3 shows the position of the seal with respect to the rotor axis and the pole of the limaçon curve. The figure also shows the angle, λ , between the rotor axis and the direction along which the housing reaction is directed as per the kinematical aspects of the limaçon design.

Figure 3: Kinematics of apex seal

The position, r_c^1 , of the seal CG with respect to the frame X_1Y_1 which is situated at the pole and rotates with the rotor is given by,

$$r_c^1 = (2r \sin \theta + L_i + x)\hat{i} + y\hat{j} \quad [15]$$

where $\hat{i}$ and $\hat{j}$ are unit vectors in the directions of X_1 and Y_1 respectively. L_i in equation [15] is the distance from the rotor centre to the initial location of the seal CG. The symbols x and y signify small deflections of the seal CG in the

direction of the axes of the frame X_1Y_1 . As the rotor rotates with a constant angular velocity, ω , the accelerations of the seal CG in the direction of X_1 and Y_1 respectively are given as follows,

$$\begin{cases} \ddot{X}_1 = \ddot{x} - 4r\omega^2 \sin \vartheta - \omega^2 L_i - \omega^2 x - 2\omega \dot{y} \\ \ddot{Y}_1 = \ddot{y} + 4r\omega^2 \cos \vartheta - \omega^2 y + 2\omega \dot{x} \end{cases}$$

[16]

where $(.)$ signifies differentiation with respect to time.

Figure 4: Seal dynamics - the elastic contact approach

The forces acting on the seal are classified as contact forces, acting where the seal contacts either the groove or the housing, and pressure forces, acting from surrounding gas chambers on the seal. The possible locations of the contact points on the seal are denoted by the letters cs_i , where $i = 1, 2, \dots, 5$. The corresponding contact points on the surrounding metal components are denoted by cm_i and contact is established if $b_i < 0$. The variable b_i corresponds to the distance from cm_i to cs_i measured in directions specified in Figure 4. If contact is confirmed at point i , the corresponding contact force, $\vec{F}_i$, is calculated as follows,

$$\vec{F}_i = (K_i |b_i| + C_i \dot{b}_i) \hat{b}_i + \mu_i (K_i |b_i| + C_i \dot{b}_i) (\hat{Z} \times \hat{b}_i)$$

[17]

where K_i and C_i are respectively the elastic and damping coefficients at contact point i ; and μ_i is the coefficient of friction at the same point. Also, $\hat{b}_i$ is a unit vector along b_i whose directions are shown in Figure 4, and $\hat{Z}$ is a unit vector perpendicular to the plane of motion. The seal is also subject to pressure forces perpendicular to its surfaces and located at points s_j where $j = 1, 2, \dots, 7$. At point s_j , the pressure force, Fp_j , is calculated as follows,

$$\overrightarrow{Fp_j} = A_j P_j \hat{f}_{s_j} \quad [18]$$

where A_j is the seal surface area near point s_j , P_j is the gas pressure near the same point and $\hat{f}_{s_j}$ signifies a unit vector inwards perpendicular to the surface whose area is A_j . The total forces, $\vec{F}$, acting on the seal is therefore calculated as follows,

$$\vec{F} = F_x \hat{X}_1 + F_y \hat{Y}_1 = \vec{F}_s + \sum_{i=1}^5 \vec{F}_i + \sum_{j=1}^7 \overrightarrow{Fp_j} \quad [19]$$

where $\vec{F}_s$ is the sum of the elastic and damping forces acting on the back end of the seal where the supporting spring is fitted.

Once all the forces have been calculated, three nonlinear differential equations are written for the system in the following form:

$$\begin{bmatrix} m_s & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & I_{CG} \end{bmatrix} \begin{bmatrix} \ddot{X}_1 \\ \ddot{Y}_1 \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ \sum M_{CG} \end{bmatrix} \quad [20]$$

where m_s is the mass of the seal and I_{CG} is the mass moment of inertia of the seal about its CG. Moreover, $\sum M_{CG}$ signifies the moment of various forces about the seal CG. In equation [20], $\ddot{\phi}$ is the angular acceleration of the seal and the two linear acceleration components, $\ddot{X}_1$ and $\ddot{Y}_1$, have been introduced in equation [16].

The three differential equations in [20] are solved numerically at every rotor angle to find the corresponding linear and angular deflections of the seal at this rotor angle. Leakage past the seal-housing gap is confirmed if the distance b_1 is found to be positive. In this case the amount of leakage is calculated by employing the orifice model to calculate the flow past the seal.

6. Dynamic response of inlet valve

To control the amount of fluid entering the machine chamber to be expanded, a control valve is fitted at the inlet as shown in Figure 5. The inclusion of an inlet valve has subsequently created an antechamber, V_3 , which separates the inlet passage and the machine working chambers. Hence, during the machine operation, fluid will need to fill this antechamber before entering the upper chamber instead of flowing directly from the inlet to the upper chamber as in

the case of no-inlet-valve design. Figure 5 shows that depends on the angular position of the rotor, then antechamber can expose itself to either the upper chamber, the lower chamber, or both at the same time. The inlet valve is of a normally open type and kept open by a spring, k_v , as shown in Figure 5. The spring force, $k_v \delta_v$, is large enough to keep the valve open even when the top surface of the valve is exposed to pressure from the supply tank. The valve stem is connected to a lever which is then connected to a plunger the motion of which controls the valve opening and closing. Movement of the plunger is controlled by two solenoids installed at the two ends of the stem (Figure 5). One solenoid helps closing the valve to cut off fluid supply, the other assists the spring during the valve opening phase. This design aims to reduce the valve response time and ensures that the machine can still functioning in case one or both of the solenoids fail. Of note is the position of the pivot point, this point is located closer to the valve stem than to the plunger; the distances from pivot point to the valve stem and plunger are a and b , respectively. The ratio of the arms will help increasing the amount of force acting on the valve stem at the same time reducing the current load on the solenoid coils.

Figure 5: Design of an inlet valve

It is now essential to derive the equation of motion of this plunger-lever-valve system. The mass of the plunger, the lever, and the valve are defined as m_1, m_2 , and m_3 respectively. The distances the valve and the plunger travel in vertical direction are z_v and x_v , respectively; linear motion of the centre of gravity (CG) of the lever in the vertical direction is y_v . The potential energy and kinetic energies of the system can be expressed as:

$$PE = \frac{1}{2}k_v(\delta_v + x_v)^2 + m_1gx_v + m_2gy_v - m_3gz_v \quad [21]$$

and

$$KE = \frac{1}{2}m_1\dot{x}_v^2 + \frac{1}{2}m_2\dot{y}_v^2 + \frac{1}{2}m_3\dot{z}_v^2 + \frac{1}{2}I_2\dot{\theta}_v^2 \quad [22]$$

Equation [21] and [22] can be rewritten in terms of z_v and $\dot{z}_v$ as follows:

$$PE = \frac{1}{2}k_v\left(\delta_v + z_v\frac{b}{a}\right)^2 + \left[m_1\frac{b}{a} + m_2\left(\frac{b-a}{2a}\right) - m_3\right]gz_v \quad [23]$$

and

$$KE = \frac{1}{2}m_1\left(\frac{b}{a}\right)^2\dot{z}_v^2 + \frac{1}{2}m_2\left(\frac{a+b}{2a}\right)^2\dot{z}_v^2 + \frac{1}{2}m_3\dot{z}_v^2 + \frac{1}{2}\frac{I_2}{a^2}\dot{z}_v^2$$

$$KE = \frac{1}{2}\left[m_1\left(\frac{b}{a}\right)^2 + m_2\left(\frac{a+b}{2a}\right)^2 + m_3 + \frac{I_2}{a^2}\right]\dot{z}_v^2 \quad [24]$$

where k_v is the spring stiffness; δ_v is the spring initial deflection. The spring is aligned with the solenoids plungers

I_2 is the moment of inertia of the lever about the pivot point

$\dot{x}_v, \dot{y}_v$, and $\dot{z}_v$ are the velocity of m_1, m_2 , and m_3 , respectively.

Differentiating the equation of potential energy with respect to z_v and kinetic energy with respect to $\dot{z}_v$ give:

$$\frac{\partial PE}{\partial z_v} = k_v\frac{b}{a}\left(\delta_v + z_v\frac{b}{a}\right) + \left[m_1\frac{b}{a} + m_2\left(\frac{b-a}{2a}\right) - m_3\right]g \quad [25]$$

and

$$\frac{\partial KE}{\partial \dot{z}_v} = \left[m_1\left(\frac{b}{a}\right)^2 + m_2\left(\frac{a+b}{2a}\right)^2 + m_3 + \frac{I_2}{a^2}\right]\dot{z}_v \quad [26]$$

Based on the Lagrange energy method, the equation of motion of this system can be written as

$$\begin{aligned} \left[m_1 \left(\frac{b}{a} \right)^2 + m_2 \left(\frac{a+b}{2a} \right)^2 + m_3 + \frac{I_2}{a^2} \right] \ddot{z}_v + c_{eq} \dot{z}_v + k z_v \left(\frac{b}{a} \right)^2 \\ = -k_v \delta_v \left(\frac{b}{a} \right) - \left[m_1 \frac{b}{a} + m_2 \left(\frac{b-a}{2a} \right) - m_3 \right] g + \Delta P C_d A_v + F_{sl} \frac{b}{a} + F_{sur} \end{aligned} \quad [27]$$

where c_{eq} is the equivalent damping coefficient;

$\Delta P = P_{in} - P_3$ is the difference in pressure between the inlet and antechamber;

C_d is the drag coefficient of the valve, $0.6 \leq C_d \leq 0.9$

A_v is the area of the vale that is subjected to the effect of pressure difference;

F_{sl} is the force exerted by the solenoids; and

F_{sur} is the contact force that the valve experiences either from the seat or from the mechanical stopper.

The force F_{sur} can be calculated by using the Algorithm 1 as follows:

Algorithm 1: Calculate the force F_{sur}

1	if $z_v > s_{st}$
2	$F_{sur} = -k_{seat}(z_v - s_{st}) - c_{seat}\dot{z}_v$
3	elseif $z_v < 0$
4	$F_{sur} = -k_{stop}z_v - c_{stop}\dot{z}_v$
5	else
6	$F_{sur} = 0$
7	end

where s_{st} is the stroke or the maximum motion of the valve stem

k_{seat} and k_{stop} are the stiffness of the seat and mechanical stopper, respectively

c_{seat} and c_{stop} are the damping coefficient of the seat and mechanical stopper, respectively

Yuan and Li (2004), in their exceptional paper, present a mathematical model from which force exerted by a solenoid can be calculated. Based on such a model, the solenoid force, F_{sl} , for this inlet valve model can be calculated as follows:

$$F_{sl} = \frac{\beta_{s1}}{2} \left(\frac{i_{s1}}{z_{s1} + d_{0s}} \right)^2 - \frac{\beta_{s2}}{2} \left(\frac{i_{s2}}{z_{s2} + d_{0s}} \right)^2 \quad [28]$$

where subscript $i = 1$ or 2 are for two-solenoid set up; i_{si} are solenoid currents;

z_{si} are the displacements of the plunger relative to the solenoids 1 and 2;

$\beta_{s1} = \beta_{s2} = \beta_s$ and d_{0s} are given by Yuan and Li (2004);

The displacement of the plunger can be calculated from the relationship with the valve stem displacement as

$z_{s1} = \frac{b}{a} z_v$ and $z_{s2} = \frac{b}{a} (s_{st} - z_v)$. The velocities $\dot{z}_{s1}$ and $\dot{z}_{s2}$ depends on the velocity of the valve as follows

$$\dot{z}_{s1} = \frac{b}{a} \dot{z}_v \quad \text{and} \quad \dot{z}_{s2} = -\frac{b}{a} \dot{z}_v \quad [29]$$

The solenoids are controlled based on the angular position of the rotor. When the inlet valve is required to be opened or closed at a certain rotor angular position, θ , a electrical voltage is applied to the opening or closing solenoids. Before the solenoids can effectively close or open the inlet valve, current flow within the coils need time to build up. This current build-up can be derived from the relationship between voltage and current as follows:

$$V_{s1} = i_{s1} R_{s1} - \frac{i_{s1} \beta_s}{(z_{s1} + d_{0s})^2} \dot{z}_{s1} + \frac{\beta_s}{z_{s1} + d_{0s}} \frac{di_{s1}}{dt} \quad [30]$$

which can be rearranged for the change of current with respect to time as:

$$\frac{di_{s1}}{dt} = \frac{z_{s1} + d_{0s}}{\beta_s} \left[V_{s1} - i_{s1} R_s + \frac{\beta_s i_{s1} \dot{z}_{s1}}{(z_{s1} + d_{0s})^2} \right] \quad [31]$$

Similarly, the build-up of current in the second solenoid can be expressed as:

$$\frac{di_{s2}}{dt} = \frac{z_{s2} + d_{0s}}{\beta_s} \left[V_{s2} - i_{s2} R_s + \frac{\beta_s i_{s2} \dot{z}_{s2}}{(z_{s2} + d_{0s})^2} \right] \quad [32]$$

The nonlinear differential equations given by [27], [31], and [32] can be solved simultaneously in an iterative fashion to calculate the valve position and the solenoid currents at every rotor angle, θ . The voltages that drive the

solenoids are given as shown in Figure 6 below to open and close the inlet valve at angles which are meant to match the power requirements of the expander for a given demand.

Figure 6: Solenoid control voltage

The effective orifice area or the equivalent area of the valve that allows the fluid to flow through during the valve opening phase can be calculated based on a simplified version of Hunt and Davis's equation as follows (Tuymmer & Machu, 2001):

$$A_{eq} = \sqrt{\frac{1}{\frac{1}{(0.85 \pi D_s (s_{st} - z_v))^2} + \frac{1}{A_v^2}}} \quad [33]$$

where $A_v = \frac{\pi}{4} D_s^2$ and D_s is the diameter of the valve seat.

The thermodynamic model of limaçon machine is detailed in the next section.

7. The thermodynamic model

The performance of a limaçon-to-circular machine depends on the derivative $\frac{dV_{net_b}}{d\theta}$, which can be expressed as:

$$\frac{dV_{net_b}}{d\theta} = 4bHL^2 \sin \theta \quad [34]$$

where V_{net_b} is volume of the working chamber below the rotor chord previously described in equation [2]. Hence, the mass flow rate, $\dot{m} = \frac{dm}{dt}$, of the fluid inside the working chamber in relation with the fluid density, the available chamber volume, and the rotor angular position can be obtained as:

$$\frac{dm}{dt} = \left(4b\rho HL^2 \sin \theta + V_{net_b} \frac{d\rho}{d\theta} \right) \frac{d\theta}{dt} = \left(4b\rho HL^2 \sin \theta + V_{net_b} \frac{d\rho}{d\theta} \right) \omega \quad [35]$$

where ρ , and m are in turn the instantaneous density and mass of the fluid inside the working chamber at any given rotor angle θ . The term $\frac{d\theta}{dt} = \omega$ is the rotor angular velocity given in $\frac{rad}{s}$. The change of the fluid mass in the working chamber below the rotor chord can be obtained by rearranging equation [35] as follows:

$$\frac{d\rho_b}{d\theta} = \frac{1}{V_{net_b}} \left(\frac{1}{\omega} \frac{dm}{dt} - 4b\rho_b HL^2 \sin \theta \right) \quad [36]$$

where the subscript b denotes “below the rotor”.

When the continuity equation is utilised and the effect of leakage through the sides of the machine and the apex seals are taken into account, equation [36] can then be re-written for the change of fluid mass in the two chambers below and above the rotor chord, respectively, as:

$$\frac{d\rho_b}{d\theta} = \frac{1}{V_{net_b}} \left(\frac{1}{\omega} (A_{ib}\rho_{idb}U_{ib} - A_{ob}\rho_{odb}U_{ob} - \dot{m}_{sb} \pm \dot{m}_{asb}) - 4b\rho_b HL^2 \sin \theta \right) \quad [37]$$

$$\frac{d\rho_a}{d\theta} = \frac{1}{V_{net_a}} \left(\frac{1}{\omega} (A_{ia}\rho_{ida}U_{ia} - A_{oa}\rho_{oda}U_{oa} - \dot{m}_{sa} \mp \dot{m}_{asa}) - 4b\rho_a HL^2 \sin \theta \right) \quad [38]$$

where subscripts i, o, d, a , and b denote “inlet”, “outlet”, “down-stream”, “above the rotor chord”, and “below the rotor chord”, respectively

$\dot{m}_s$ and $\dot{m}_{as}$ are the mass flowrate of leakages of working fluid through the sides of the machine and apex seals, respectively

The velocity of the fluid through the inlet and outlet ports, U_i and U_o , can be calculated by utilising equation [13]. The signs $\pm$ show the directions of such leakages, which are governed by the instantaneous value of pressures in the upper and lower chambers.

When the energy transfers to and from the fluid inside the working chamber is considered to occur as an adiabatic process, the following equation can be utilised:

$$\frac{dh_i}{dt} - \frac{dh_o}{dt} = \frac{d(me)}{dt} + \frac{PdV}{dt} \quad [39]$$

where h_i , and h_o are the total enthalpy moving in and out of a working chamber, respectively; e is the specific internal energy available in the working chamber. Since, the term internal energy, e , can be substituted by $h - pv$, and $m \frac{dh}{dt} = mT \frac{ds}{dt} + V \frac{dP}{dt}$, equation [39] can be re-written as:

$$\frac{dh_i}{dt} - \frac{dh_o}{dt} = \frac{mTds}{dt} + \frac{hdm}{dt} \quad [40]$$

or

$$mT\omega \frac{ds}{d\theta} = (h_i - h) \frac{dm_i}{dt} - (h_o - h) \frac{dm_o}{dt} \quad [41]$$

The above equation can then be manipulated to produce equations, which show the rate of entropy change in the two working chambers per rotor angular rotational angle as below:

$$\frac{ds_b}{d\theta} = \frac{1}{\omega \rho_b V_b T_b} [A_{ib} U_{ib} \rho_{idb} (h_{iub} - h_b) - A_{ob} U_{ob} \rho_{odb} (h_{oub} - h_b) - \dot{m}_{sb} (h_{sb} - h_b) \pm \dot{m}_{asb} (h_{asb} - h_b)] \quad [42]$$

$$\frac{ds_a}{d\theta} = \frac{1}{\omega \rho_a V_a T_a} [A_{ia} U_{ia} \rho_{ida} (h_{iua} - h_a) - A_{oa} U_{oa} \rho_{oda} (h_{oua} - h_a) - \dot{m}_{sa} (h_{sa} - h_b) \mp \dot{m}_{asa} (h_{asa} - h_a)] \quad [43]$$

where the subscripts i, o, d, u, a , and b denote “inlet”, “outlet”, “down-stream”, “up-stream”, “above the rotor chord”, and “below the rotor chord”, respectively

$\dot{m}_s$ and $\dot{m}_{as}$ are the mass flowrate of leakages of working fluid through the sides of the machine and apex seals

h_s and h_{as} are the enthalpy on the upstream side of the sides and apex seals

Equations [37], [38], [42], and [43] can be solved simultaneously to find the instantaneous values of densities, and entropies below and above the rotor chord, ρ_b, ρ_a, s_b , and s_a , of fluid in the two working chambers at each crank angle, θ . At each calculation step, the corresponding values of pressure, P , density, ρ , and viscosity, μ , can be calculated using the above equations. The instantaneous values for temperature, T , and enthalpy, h , can be calculated with the assistance of the functions $T(P, s)$, and $h(P, s)$ offered in REFPROP, which can be linked to

Matlab for fluid properties calculation (Lemmon, Huber, & McLinden, 2010; National Institute of Standards and Technology, 2016). The REFPROP functions deliver the fluid properties for not only single but also two-phase flows that are accurate enough for the calculations in this paper. In order to achieve further accuracy for the speed of sound in two-phase flow, the REFPROP functions has been utilised in conjunction with the algorithm offered in the published work by Lund and Flatten (2010).

In this paper, the authors investigate one of the embodiments of the limaçon machines; hence, the instantaneous crankshaft torque value can be calculated based on the equation suggested by Sultan (2005) as shown below:

$$\tau = 4bL^2 H(P_{cb} - P_{ca}) \sin \theta \quad [44]$$

where the subscript c, b , and a are “chamber”, “below the rotor”, and “above the rotor” respectively.

The calculation process for the machine crankshaft torque has been approached iteratively by applying the thermodynamic model and compute at small crank angle increments within the range $\theta_{li} \leq \theta \leq \theta_{li} + \pi$ in which the θ_{li} is angular position of the inlet port leading edge. The values of pressure and density below the rotor at the start of the cycle, $P_{cb}(\theta_{li})$ and $\rho_{cb}(\theta_{li})$, respectively, are compared with the corresponding values of pressure and density above the rotor chord at the end of the cycle, $P_{ca}(\theta_{li} + \pi)$ and $\rho_{ca}(\theta_{li} + \pi)$. At the same time, the values of pressure and density above the rotor at the start of the cycle, $P_{ca}(\theta_{li})$ and $\rho_{ca}(\theta_{li})$, respectively, are compared with the corresponding values of pressure and density below the rotor at the end of the cycle, $P_{cb}(\theta_{li} + \pi)$ and $\rho_{cb}(\theta_{li} + \pi)$. The comparison process is done by using the dimensionless error expression shown below:

$$\epsilon = \left[\left(\frac{P_{cb}(\theta_{li}) - P_{ca}(\theta_{li} + \pi)}{\bar{P}_l} \right)^2 + \left(\frac{\rho_{cb}(\theta_{li}) - \rho_{ca}(\theta_{li} + \pi)}{\bar{\rho}_l} \right)^2 + \left(\frac{P_{ca}(\theta_{li}) - P_{cb}(\theta_{li} + \pi)}{\bar{P}_u} \right)^2 + \left(\frac{\rho_{ca}(\theta_{li}) - \rho_{cb}(\theta_{li} + \pi)}{\bar{\rho}_u} \right)^2 \right]^{1/2} \quad [45]$$

where $\bar{P}_l = \frac{P_{cb}(\theta_{li}) + P_{ca}(\theta_{li} + \pi)}{2}$, $\bar{\rho}_l = \frac{\rho_{cb}(\theta_{li}) + \rho_{ca}(\theta_{li} + \pi)}{2}$, $\bar{P}_u = \frac{P_{ca}(\theta_{li}) + P_{cb}(\theta_{li} + \pi)}{2}$, and $\bar{\rho}_u = \frac{\rho_{ca}(\theta_{li}) + \rho_{cb}(\theta_{li} + \pi)}{2}$ are the pressure and density coefficients used to calculate the error.

If the value of error expression, ϵ , is larger than a small value defined by the designer, the values of $P_{cb}(\theta_{li})$, $\rho_{cb}(\theta_{li})$, $P_{ca}(\theta_{li})$, and $\rho_{ca}(\theta_{li})$ are set to be equal to $P_{ca}(\theta_{li} + \pi)$, $\rho_{ca}(\theta_{li} + \pi)$, $P_{cb}(\theta_{li} + \pi)$

π), and $\rho_{cb}(\theta_{li} + \pi)$, respectively. The calculation process is then repeated over the range of $\theta \in [\theta_{li}, \theta_{li} + \pi]$ until the outcome of the error expression falls below the predefined value, which reflects the cyclical nature of the thermodynamic process. Once the convergence is achieved, the total energy per one cycle, E_{cycle} , can be calculated by utilising the following equation:

$$E_{cycle} = 2 \delta\theta \left(\sum_{n=1}^N \tau_{sn} - \frac{\tau_{sn} + \tau_{s1}}{2} \right) \quad [46]$$

where $\delta\theta$ is the size of the angular interval, $n \in [1, N]$ is the counter for the successive points on the torque curve, and N is the total number of intervals on the torque curve.

With the same approach, the total mass flow through the machine in one cycle, M_{cycle} , can be calculated using the expression below:

$$M_{cycle} = 2 \frac{\delta\theta}{\omega} \left\{ \sum_{n=1}^N \left[(A_{ib}\rho_{idb}U_{ib})_n + (A_{ia}\rho_{ida}U_{ia})_n \right] - \frac{(A_{ib}\rho_{idb}U_{ib})_n + (A_{ib}\rho_{idb}U_{ib})_1}{2} - \frac{(A_{ia}\rho_{ida}U_{ia})_n + (A_{ia}\rho_{ida}U_{ia})_1}{2} \right\} \quad [47]$$

The machine overall efficiency can be calculated from the total energy per one cycle and total mass flow through the machine as follows:

$$\eta_o = \frac{E_{cycle}}{M_{cycle}\Delta h_{is}} \quad [48]$$

where Δh_{is} is the difference between the enthalpy in the inlet manifold and its isentropically reduced value in the discharge manifold. It should be highlighted that in order to account for the fact that for each full rotation of the shaft, the limaçon-to-circular machine has two fluid expanding cycles, hence, a multiplication by two is incorporated in equations [46] and [47].

8. Numerical illustrations

The study of this limaçon-to-circular expander utilises organic gas R-245fa as a working fluid. Saturated R-245fa is made available at an absolute pressure of 10 bars and temperature of 145°C. The working fluid will be expanded from 10 bars to the pressure of 1 bar at the outlet manifold. The expander is allowed to run at various angular velocity i.e.: 800, 1200, 1500, and 1800 RPM and various values of antechamber, V_3 . Some of the machine dimensions such as rotor half-chord length, L , limaçon aspect ratio, b , limaçon-to-circular design factor, a , radial clearance, Δ_{min} , rotor depth multiplication factor, β , and clearance volume factor, μ , will be used as constant in this study. These dimensions, which have been previously optimised for limaçon-to-circular machine (Phung, Sultan, & Boretti, 2016), are shown in Table 1 below. The limaçon rotor axial depth is given as $H = \beta L$.

Table 1: Dimensions of the limaçon-to-circular machine

	Values	Units
Rotor half-chord length, L	64.29	mm
Limaçon aspect ratio, b	0.165	-
The design factor, a	1.096	-
Radial clearance, Δ_{min}	0.1	mm
Rotor depth multiplication factor, β	1.3	-
Clearance volume factor, μ	0.167	-

In the following table, the values for inlet and outlet port angular positions, angular widths, and lengths are set as:

Table 2: Limaçon machine's port geometries

	Values
Inlet port angular position, θ_{li}	-25°
Outlet port angular position, θ_{lo}	140°
Inlet port angular width, $\Delta\theta_i$	31.78°
Outlet port angular width, $\Delta\theta_o$	32.9°
Inlet port length, L_o	71 mm
Outlet port length, L_i	107 mm

A representation of chamber pressure and temperature against the crank angle, θ , and the pressure-volume relationship can be found in Figure 7 and Figure 8 as below:

Figure 7: Chamber pressure and temperature against crank angle

Figure 8: Limaçon machine PV diagram

Volume of the antechamber, V_3 , also affect the pressure within the machine working chamber as shown in Figure 9. When volume of antechamber is small, pressure inside the machine working chamber tends to drop further compared to the case where antechamber volume is large. This is also true with the temperature drop in the working chamber (Figure 9). The larger the antechamber gets, fluid supply to the limaçon machine is more sufficient which can help the machine utilising the pressure difference between the inlet and outlet more effectively. Moreover, increasing the antechamber volume, of course to a certain extent, also means that the inlet valve doesn't need to stay open for a long period of time to supply working fluid; this effect will be shown later in this section.

Figure 9: The effect of antechamber volume to the working chamber pressure at various RPM

The magnitude of solenoid voltages V_{s1} and V_{s2} can be set by the designer based on the operational condition of limaçon machine; here, these voltages were set equal as $V_{s1} = V_{s2} = V_s = 75 \text{ Volt}$. The valve opening and closing

with respect to rotor angular displacement at different RPMs and antechamber volumes are shown in Figure 10; of note is that the valve stroke is set at 5 mm (Figure 10).

Figure 10: Valve opening against rotor angle at various RPM and V_3

In this design, the valve is fully open when valve motion, $z_v = 0$; when $z_v = 5 \text{ mm}$, the valve is fully closed. Figure 10 shows the valve action with the help from solenoids at various angular velocities. It is worth noting that when voltage is supplied to the coils, the solenoid currents need to build-up before the solenoids can jump into action to close the valve and cut off fluid supply or assist the valve spring in the valve opening phase. Such current build-ups are shown in Figure 11. Obviously, the valve displacement, z_v , increases or decreases gradually according to solenoid responses. Figure 10 also shows that the valve can stay open for a much longer period at lower RPM – shown as solid blue curve – compared to higher RPM – shown as red or solid black curve. This results in better fluid supply for the antechamber and consequently the machine working chamber. The solid magenta curve shown in Figure 10, in particular, is an indication that the inlet valve cannot achieve fully close position. This is due to the relatively slow response time of the solenoids compared to the machine angular velocity. This problem can be easily overcome when faster response time solenoids are used in limaçon machines that operate at higher RPM.

Figure 11: Solenoid currents at one particular RPM

Figure 12 and Figure 13 show linear and angular magnitudes of the seal motion as well as the seal-housing forces at 800 and 1800 RPM, respectively. The forces exerted between the seal and housing when the machine runs at 1800 RPM is significantly higher than that of the 800 RPM. In fact, this behaviour is expected since at lower RPM the difference in pressure between the machine chamber and the outlet is much greater as shown in Figure 9. This pressure difference tends to push the seal against one side of the seal groove and help reducing both linear and angular

displacement of the seal. At higher RPM, the centrifugal force on the seal together with the lower in pressure difference have seen seal motion inside the seal groove and seal-housing force increasing. Such an increase in seal-housing force will result in higher friction force between the seal tip and the machine housing surface which will adversely affect the machine performance. Hence it is critical to find an optimum seal spring stiffness, inlet valve settings, volume of antechamber, and RPM to maximise the machine output power.

Figure 12: Seal displacements and seal-housing force @800 RPM and $V_3 = 2.85 V_{max}$

Figure 13: Seal displacements and seal-housing force @1800 RPM and $V_3 = 2.85 V_{max}$

The overall efficiencies of the limaçon machine at different RPM and values of antechamber V_3 , as shown in Figure 14, can be calculated using equations [46], [47], and [48]. It is worth noting that although the amount of friction loss between seal and housing increases with the increase

of RPM, the overall efficiency of the machine improves significantly at higher speed.

Figure 14: Limaçon machine overall efficiencies at different RPM and V_3

9. Conclusion

This paper investigates the thermodynamic performance of the limaçon-to-circular machines; the inlet and outlet ports' angular positions, angular widths, and lengths of the limaçon-to-circular machines have been previously optimised based on the operational condition. The seal dynamic models as well as the inlet valve dynamics response have been taken into account to demonstrate and compare the effectiveness and efficiency of the machine due to sealing and inlet valve inclusion. Future work on limaçon machine optimisation can be done with the incorporation of apex seal and inlet valve dynamics.

10. Nomenclatures

$2L$: the limaçon chord length

XY : stationary frame

$X_r Y_r$: moving frame

o : limaçon pole

p_l, p_t : leading and trailing apices

θ : crank angle

r : radius of the base circle

m : centre point of the chord

R : radius of the circular segment

C : clearance value

b : limaçon aspect ratio

V_{net} : volume of the working chamber

H : the expander's housing depth

$\mathbf{R}_l, \mathbf{R}_t$: position vector of the ports' leading and trailing edges

$\hat{\mathbf{R}}_l, \hat{\mathbf{R}}_t$: leading and trailing unit vectors

θ_l, θ_t : angular positions of the leading and trailing edges

L_p : port length

W : port width

A_f : cross sectional area of the port

A_p : instantaneous value of the port area

P_l, P_t : position of the rotor's leading and trailing edges

$\hat{\mathbf{X}}_r$: unit vector along the rotor chord

$\hat{\mathbf{Y}}_r$: unit vector along the axis perpendicular to the rotor chord

s_l, s_t : relative locations of the rotor leading and trailing apices with respect to the port leading and trailing edges

U : velocity of the fluid

h_u, h_d : upstream and downstream isentropic enthalpy

Δh : isentropic enthalpy drop along the flow direction

K_p : the loss coefficient

f : friction factor

Re : Reynolds number

ρ : density of the working fluid

μ : viscosity

x : dryness fraction

$\dot{m}$: mass flow rate

τ : crankshaft torque

ϵ : error expression used to end the cyclical iteration

Φ : design vector

E_{cycle} : total energy per cycle

$\delta\theta$: size of the angular interval

M_{cycle} : total mass flow per cycle

ω : angular velocity of the rotor

m_1, m_2, m_3 : mass of the plunger, the lever, and the inlet valve

k_v, k_{stop}, k_{seat} : stiffness of the valve spring, the mechanical stopper, and the seat

c_{seat}, c_{stop} : damping coefficient of the seat and mechanical stopper

δ_v : valve spring initial deflection

I_2 : moment of inertia of the lever about the pivot point

x_v, y_v, z_v : displacement of the plunger, lever, and valve

$\dot{x}_v, \dot{y}_v, \dot{z}_v$: velocity of the plunger, lever, and valve

$\ddot{z}_v$: acceleration of the valve

F_{sl} : force exerted by the solenoids

F_{sur} : contact force that the valve experiences

V_3 : volume of the antechamber

ΔP : difference in pressure between the inlet and antechamber

A_v : area of the valve that is subjected to the effect of pressure difference

i_{s_i} : solenoid currents

V_{s_i} : solenoid supplied voltage

z_{s_i} : displacements of the plunger relative to the solenoids 1 and 2

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