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Steering Approaches to Pareto-Optimal Multiobjective Reinforcement Learning

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Abstract

For reinforcement learning tasks with multiple objectives, it may be advantageous to learn stochastic or non-stationary policies. This paper investigates two novel algorithms for learning non-stationary policies which produce Pareto-optimal behaviour (w-steering and Q-steering), by extending prior work based on the concept of geometric steering. Empirical results demonstrate that both new algorithms offer substantial performance improvements over stationary deterministic policies, while Q-steering significantly outperforms w-steering when the agent has no information about recurrent states within the environment. It is further demonstrated that Q-steering can be used interactively by providing a human decision-maker with a visualisation of the Pareto front and allowing them to adjust the agent's target point during learning. To demonstrate broader applicability, the use of Q-steering in combination with function approximation is also illustrated on a task involving control of local battery storage for a residential solar power system.

Keywords: multiobjective reinforcement learning, non-stationary policies, geometric steering, interactive reinforcement learning, Pareto optimality

1. Introduction

Reinforcement learning (RL) methods learn the optimal behaviour for an agent on the basis of a reward signal received from the agent's environment. While most RL research assumes the agent has only a single objective

5 to be maximised, many sequential decision-making tasks exhibit multiple
6 conflicting objectives. Examples include managing competing uses of water
7 reservoirs [1], reducing energy usage and increasing miss probability detection
8 in cognitive radio networks [2], achieving both optimal control and stability
9 in a control system [3], and considering both engagement time and transi-
10 tion smoothness in wet clutch control [4]. Therefore recent years have seen
11 multiobjective reinforcement learning (MORL) emerge as a growing area of
12 research. For an overview of this field see [5].

13 The difference between single-objective RL and MORL is straightforward
14 – rather than a scalar reward, the agent receives a vector reward with an
15 element for each objective. For MORL problems multiple Pareto-optimal
16 policies may exist, and the agent must either learn several of these policies, or
17 identify a single policy which best matches a pre-specified tradeoff between
18 the objectives. It may be that no deterministic stationary policy closely
19 matches the desired trade-off, and so non-stationary or stochastic policies
20 may be superior. However there has been relatively little work so far on
21 MORL algorithms using such policies.

22 This paper addresses this limitation by investigating two novel algorithms
23 for learning non-stationary policies for multiobjective problems, w-steering
24 and Q-steering¹. These algorithms adapt the Multiple Direction Reinforce-
25 ment Learning (MDRL) steering algorithm of [7, 8] to the task of learning
26 Pareto-optimal policies. w-steering is a direct extension of MDRL, while
27 Q-steering incorporates a heuristic policy-selection mechanism to address
28 MDRL’s reliance on pre-existing knowledge of the identity of recurrent states
29 of the environment. The performance of these two approaches to Pareto-
30 optimal steering are empirically compared, demonstrating Q-steering’s supe-
31 rior performance across a variety of problems.

32 This paper then examines two extensions of Q-steering. Section 7 pro-
33 poses an interactive approach to MORL, by providing a human user with a
34 visualisation of the Pareto front learnt by Q-steering, and allowing them to
35 iteratively modify the agent’s behaviour. Section 8 demonstrates the applica-
36 bility of Q-steering to real-world problems with high-dimensional state-space

¹Some preliminary results related to these algorithms were presented in [6]. This paper provides an extended justification for the design of these algorithms, and an enhanced experimental evaluation of the factors affecting their performance, as well as considering extensions of the Q-steering algorithm to interactive MORL and learning in state spaces requiring function approximation.

37 by using Q-steering in conjunction with function approximation to control
 38 local battery storage for a photovoltaic power system.

39 2. Background

40 Single-objective RL generally assumes that the environment can be mod-
 41 elled as a *Markov decision process* (MDP) defined by the following variables:

- 42 • S is a finite set of *states*,
- 43 • A is a finite set of *actions*,
- 44 • $T : S \times A \times S \rightarrow [0, 1]$ is a *state transition function* specifying for each
 45 state, action, and next state, the probability of that next state occurring,
- 46 • $R : S \times A \times S \rightarrow \mathfrak{R}$ is a *reward function*, which defines the expected
 47 immediate reward for each state, action and subsequent state,
- 48 • $\mu : S \rightarrow [0, 1]$ is a probability distribution over initial states, and
- 49 • $\gamma \in [0, 1)$ is a *discount factor* specifying the relative importance of short-
 50 term and long-term rewards.

The goal of the agent is to find a policy (a mapping from state to action) which maximizes the expected *return* R_t . In a *finite horizon (or episodic)* MDP, the return is typically an undiscounted finite sum of the rewards over all time-steps until a terminating state is reached, while in an *infinite horizon* MDP, the return is usually an infinite sum, with each term discounted according to γ :

$$R_t = \sum_{k=0}^{\infty} \gamma^k r_{t+k+1},$$

51 where r_t is the reward obtained at time t .

In *multiobjective MDPs* (MOMDPs) the reward function $\mathbf{R} : S \times A \times S \rightarrow \mathfrak{R}^n$ is a vector of n rewards rather than a scalar, with an element for each objective. Similarly, the return is also a vector value. For example, the multiobjective infinite horizon return $\mathbf{R}_t$ would be defined as:

$$\mathbf{R}_t = \sum_{k=0}^{\infty} \gamma^k \mathbf{r}_{t+k+1},$$

52 For single objective MDPs there will be either a single optimal policy, or
 53 a set of optimal policies with identical returns. For MOMDPs a policy can

54 outperform another on one objective, but be inferior with regards to the other
 55 objective(s). This is summarized by the Pareto dominance relationship. A
 56 policy dominates another if it is superior on at least one objective, and at
 57 least equal on all other objectives. Clearly a dominating policy is preferable
 58 to the policies it dominates. However there may be many policies which are
 59 non-dominated - all such policies may be optimal, depending on the desired
 60 trade-off between the objectives. [9, 5] propose that MORL algorithms can
 61 be divided into two broad categories:

- 62 • *multi-policy* approaches that learn multiple policies which approximate the
 63 complete Pareto front, or a subset of that front; or
- 64 • *single-policy* approaches that learn a single policy which best satisfies a
 65 desired trade-off between objectives expressed in advance.

66 As well as differentiating between multiple optimal policies, a MORL
 67 agent may need to consider forms of policies which are not required in a
 68 single-objective context. For this work we consider the following policy types:

- 69 • *deterministic stationary* (DS) policies always select the same action when
 70 faced with a particular state of the environment. The subset of DS policies
 71 which can be found via linear scalarisation (see Section 3) is also relevant
 72 – we will refer to these as *linear deterministic stationary* (LDS) policies,
- 73 • *stochastic* policies in which the choice of action is non-deterministic, and
- 74 • *non-stationary* policies in which the choice of action in any state is non-
 75 stationary with regards to the state, and is conditioned on some other
 76 variable, such as the rewards previously received by the agent.

Figure 1: An MOMDP with 3 actions, labeled with their rewards. The graph shows average per-action reward for each deterministic stationary policy π_1 , π_2 and π_3 and for a stochastic policy π_s which selects actions a_1 and a_2 with equal probability.

77 For fully-observable single-objective MDPs there is always a DS pol-
 78 icy which is optimal [10]. However this may not be true for MOMDPs,

where stochastic or non-stationary policies may dominate the best DS policies [11, 12]. Consider the single-state MOMDP in Figure 1. Only three DS policies exist, all of which are non-dominated. If a balanced trade-off between objectives is required then policy π_3 which always selects a_3 will be the best option. However consider a stochastic policy π_s which selects actions a_1 and a_2 with probabilities p_1 and $(1 - p_1)$ respectively. The average reward of this policy will be $(3p_1, 3 - 3p_1)$. If $1/3 \leq p_1 \leq 2/3$ then this policy will dominate the DS policy π_3 . This simple example illustrates the general point that for multiobjective problems it may be possible to generate superior results using policies which are either non-stationary or stochastic.

This section has outlined the general manner in which MORL differs from the more traditional single-objective case. For the remainder of this paper it is assumed that the MORL agent must identify a single Pareto-optimal policy that satisfies the user’s preferred trade-offs between objectives, and that it will exploit the benefits of stochastic or non-stationary policies. In addition it is assumed that the algorithm must be well-suited to online learning, and that it will not require any prior knowledge of the MOMDP’s state transition or reward dynamics. The implications of these requirements will be explored in the next two sections which review existing approaches to MORL with stochastic and non-stationary policies.

3. Learning Stochastic or Non-Stationary Multiobjective Policies

The MORL literature has explored two main approaches to learning non-stationary or stochastic policies. The first uses policy search methods which directly produce stochastic policies, such as policy-gradient methods [10, 13] or multiobjective evolutionary algorithms [14, 15]. Policy search methods may have advantages for MORL, as they can be applied in conjunction with non-linear scalarisation functions to specify desired trade-offs between objectives [5]. However comparisons of policy search and value-based single-objective RL indicate that policy search algorithms may converge more slowly and so be less suitable for on-line learning, particularly if state transitions or rewards are stochastic. [16, 17, 18]. As such there may be situations in which a value-based approach to stochastic or non-stationary MORL is preferable.

The main value-based approach explored is to find DS policies, and then to form stochastic or non-stationary combinations of these ‘base policies’ [7, 8, 12, 19]. An advantage of this approach is that the base policies can be found using simple methods. For example, linearly scalarised temporal

115 difference learning has been widely used to find LDS policies for MORL
 116 tasks [20, 21, 22]. Linear scalarisation takes a weighted sum of the rewards,
 117 converting the problem to a single-objective MDP so standard TD-based
 118 methods can be used [5]. Repeated runs with differing weights can discover
 119 a range of Pareto-optimal policies [1]². The agent then switches between
 120 base policies, either stochastically [12] or on a non-stationary basis [7, 8].
 121 An advantage of these methods is that once the base policies have been
 122 learnt they can readily be combined to produce different stochastic or non-
 123 stationary policies with no further sample costs. This paper extends the work
 124 of [7, 8], and so the next section will discuss their approach in more detail.

125 4. The Geometric Steering Approach to MORL

126 [7] proposed and analysed an MORL algorithm based on the concept of
 127 geometric steering. [8] provided a more detailed presentation and formally
 128 named it Multiple Directions Reinforcement Learning (MDRL). MDRL is de-
 129 signed for stochastic games and so assumes that the environment may also be
 130 influenced by the actions of another agent. However [7] note that MOMDPs
 131 can be treated as a special case of this multiobjective stochastic game model.
 132 We will restrict our discussion of MDRL to the MOMDP scenario.

133 As outlined in Algorithm 1, MDRL forms a non-stationary policy condi-
 134 tioned on prior rewards, which aims to keep the average reward vector within
 135 a defined target region in objective space. The agent learns in parallel mul-
 136 tiple base policies based on linear scalarisation with different weight vectors.
 137 At any time a single base policy is active and the agent ‘steers’ the average
 138 reward towards the target region by switching between base policies. The
 139 choice of active policy is based on a direction vector from the average reward
 140 vector obtained so far to the closest point in the target region. The policy
 141 whose weights best match the direction vector is then selected as the active
 142 policy (see lines 8 and 9 of Algorithm 1). The intuition here is that following
 143 this policy will drive the average reward vector towards the target region.

144 MDRL has a number of desirable properties. The desired trade-off be-
 145 tween objectives is specified in a straight-forward fashion via the bounds

²As demonstrated by [23] linear scalarisation is limited to discovering policies which lie on the convex hull of the Pareto front (the Convex Coverage Set or CCS [19]). However in the context of learning base policies this is actually advantageous as these CCS policies are in fact sufficient to construct all optimal stochastic or non-stationary policies [12, 19].

146 of the target region. In addition, steering can be viewed as a flexible ‘meta-
 147 algorithm’ as the learning of base policies can be achieved using any methods
 148 based on linear scalarisation of the objectives – while the results reported in
 149 this paper are based on Q-learning, any off-policy algorithm could be used.
 150 Steering can also be applied in non-learning contexts, to form a combination
 151 of base policies found via multiobjective planning methods such as [19, 24].

Algorithm 1 The Multiple Directions Reinforcement Learning algorithm

input: target region T , set of recurrent states S_R , number of base policies j

- 1: initialize weight-vectors $w_1..w_j$ to be evenly distributed across the surface of a unit hypersphere with dimensionality equal to the number of objectives
- 2: initialise state-action values Q for j different deterministic policies $\pi_1.. \pi_j$
- 3: initialise the average reward vector R to the null vector
- 4: select at random an active policy π_A
- 5: observe initial state s
- 6: **while** true **do**
- 7: **if** s in S_R and R is not in T **then**
- 8: let D = the vector from R to the closest point in T
- 9: select active policy π_A where $A = \arg \min_{1 \leq A \leq j} \| D - w_A \|_2$
- 10: **end if**
- 11: execute action based on π_A , observe reward r and next state s'
- 12: update record of average reward R
- 13: update values for Q_{π_A}
- 14: update values for all other Q_{π_i} using off-policy learning
- 15: $s \leftarrow s'$
- 16: **end while**

152 However MDRL relies on two key assumptions. It is assumed that one
 153 or more states of the environment are recurrent – that is, they are guaran-
 154 teed to be revisited within a finite period of time regardless of the policy
 155 followed by the agent. Formally, if the hitting time of a state s^* is defined as
 156 $\tau_{s^*} = \min\{n > 0 : s_n = S^*\}$ and E_π^s is the expectation operator when start-
 157 ing from state $s_0 = s$ and following policy π , then there must exist at least
 158 one state s^* and a finite constant N such that $E_\pi^s(\tau_{s^*}^2) < N$ for all $\pi \in \Pi$
 159 and $s \in S$. In addition it is assumed that the agent knows the identity of
 160 these recurrent states and only switches policies when in one of these states,

161 as indicated by the set S_R in Algorithm 1. This requires prior knowledge of
 162 the environment’s state dynamics. For some problems this may be easy – for
 163 example in episodic problems the initial state of each episode can be treated
 164 as recurrent. However more generally there may be insufficient prior knowl-
 165 edge to identify these states, and then the only viable option is for MDRL to
 166 allow switching in any state, which could lead to suboptimal performance³.
 167 [8] address this limitation in an alternative algorithm known as Current
 168 Direction Reinforcement Learning (CDRL). CDRL maintains a single pol-
 169 icy, which has scalarisation weights set to match the current direction from
 170 the average reward vector to the target set. This policy is followed for an
 171 extended period of time before considering whether to switch policy by al-
 172 tering the weights. Switching can occur in any state, and so CDRL does not
 173 require S_R as an input. By following a single policy for an extended period,
 174 CDRL reduces the frequency of policy switching which will minimise, but not
 175 eliminate, the potential impact of switching policies in inappropriate states.
 176 However while CDRL will maintain the long-term average reward within the
 177 target region, in the short-term the average reward may lie well outside this
 178 region. For many applications this increase in the variance of the short-term
 179 reward will be unacceptable. For this reason we will not consider CDRL fur-
 180 ther in this work. Instead the next section will explore two variations of the
 181 MDRL approach to steering, including one specifically designed to address
 182 the problems introduced by lack of information about recurrent states.

183 5. Applying Steering to Pareto-Optimal MORL

184 While MDRL provides a means for learning non-stationary policies for
 185 MOMDPs, there are several differences between the context for which it
 186 was originally designed, and the context which this work addresses. Most
 187 significantly, MDRL treats the vector rewards as criteria to be satisfied rather
 188 than as objectives to be maximised. As shown in Figure 2 the specified
 189 target region of reward space may include both lower and upper bounds
 190 for each reward. For example the scenario in [7] requires the agent to keep
 191 the environmental temperature and humidity within a bounded region. The
 192 agent may need to steer in both positive and negative directions in reward
 193 space (as shown by vectors D_1 to D_4). Therefore the agent must identify

³The alternative is to treat no states as recurrent, but this would mean the agent would never switch base policies, thereby losing the benefits of non-stationarity.

194 not just the Pareto-optimal policies (π_1 and π_2) but also the dominated
 195 policies with rewards lying on the other side of the target region (π_3 and π_4).
 196 To achieve this, the weight vectors $w_1..w_j$ must include both positive and
 197 negative weights. In contrast when the goal is to produce Pareto-optimal
 198 behavior (as in Figure 3), the use of negative weights is neither required nor
 199 appropriate.

Figure 2: Illustrating MDRL. T is the target region, the white dots R_1 to R_4 indicate possible points for the average reward so far, and the arrows D_1 to D_4 indicate the corresponding direction vectors to the nearest point in T . The black dots π_1 to π_4 indicate the base policies which may be followed by the agent.

Figure 3: Pareto-optimal steering. T is the target point, the white dots R_1 to R_4 indicate possible points for the average reward so far, and the arrows D_1 to D_4 indicate the direction vectors to T . In contrast to Figure 2 the agent will only require the Pareto-optimal base policies π_1 and π_2 .

200 Similarly in the context of Pareto optimality, there may be no pre-existing
 201 knowledge on which to base the size or shape of the target region, so we
 202 propose to use a target point in objective space instead, as shown in Figure

203 3. Specifying a target point is intuitive for a human decision-maker, and
 204 by changing the target, the agent can be directed towards different Pareto-
 205 optimal policies. In addition the user may interactively change the target
 206 during learning as they learn more about the achievable trade-offs between
 207 the objectives – this approach will be examined further in Section 7.

Algorithm 2 The w-steering algorithm for Pareto-optimal MORL

input: target point T , number of base policies j , set of switching states S_S
 which will be S_R if knowledge exists of recurrent state(s) or S otherwise,
 1: initialize weight-vectors $w_1..w_j$ to be evenly distributed within the positive
 quadrant of a unit hypersphere with dimensionality equal to the
 number of objectives
 2: initialise state-action values Q for j different deterministic policies $\pi_1..\pi_j$
 3: initialise the average reward vector R to the null vector
 4: select at random an active policy π_A
 5: observe initial state s
 6: **while** true **do**
 7: **if** s in S_S **then**
 8: let $D = T - R$
 9: select active policy π_A where $A = \arg \min_{1 \leq A \leq j} \| D - w_A \|_2$
 10: **end if**
 11: execute action based on π_A , observe reward r and next state s'
 12: update record of average reward R
 13: update values for Q_{π_A}
 14: update values for all other Q_{π_i} using off-policy learning
 15: $s \leftarrow s'$
 16: **end while**

208 Algorithm 2 modifies MDRL to address the goal of finding a Pareto-
 209 optimal policy. However this modified algorithm retains the fundamental
 210 assumption of MDRL – that the agent is provided with a set of states which
 211 are known to be recurrent under all policies. Both algorithms may perform
 212 poorly in the absence of such knowledge as their policy selection (line 9 of
 213 Algorithms 1 and 2) is based on the direction vector to the target and the
 214 weight vector of each base policy, and so is independent of the current state
 215 of the environment. This fails to take into account that certain states may be
 216 far more desirable for some base policies than for others – for example in the
 217 Linked Rings problem (Figure 6 in Section 6) states 2, 3 and 4 are desirable

218 if maximizing the first objective, but undesirable if maximizing the second
 219 objective. We refer to this as the *switching-state identification* problem.

Figure 4: (left) A simple environment to illustrate a potential issue with weight-based policy selection as used in MDRL and Algorithm 2.

220 The MDRL approach to policy selection has a further potential flaw.
 221 Selecting the policy with weights that best align with the direction vector D
 222 may not in fact result in moving the average reward closer to the target, due
 223 to the non-linear relationship between the weights and the return achieved by
 224 the policy [25]. Consider the example in Figure 4. If the base policy weights
 225 are $w_1 = (1,0)$, $w_2 = (\sqrt{0.5}, \sqrt{0.5})$, and $w_3 = (0,1)$, then three base policies
 226 π_1 , π_2 and π_3 will be discovered corresponding to always selecting actions
 227 a_1 , a_2 and a_3 respectively (action a_4 is Pareto-dominated by a_1 and so is not
 228 selected under any Pareto-optimal policy). Assume that the target is $(50,30)$,
 229 which can be achieved by alternating between π_1 and π_2 . Also assume that
 230 due to exploration the agent has so far selected actions a_1 , a_2 , a_3 and a_4 in
 231 the ratio 7:7:2:4, giving rise to an average reward of $(48.5, 26)$. The vector
 232 D will be $(6.5, 4)$, so the active policy at this point should favour the first
 233 objective over the second (i.e. π_1). However Algorithm 2 will select policy π_2
 234 as its weight vector is the smallest distance from D . In the context of steering
 235 towards a large target region as in MDRL, this issue is less problematic –
 236 however in the context of steering towards a target point, policy selection
 237 errors such as this will lead to behaviour which is not Pareto-optimal. We
 238 will refer to this as the *policy identification* problem.

239 We propose a modified Pareto-optimal steering algorithm to account for
 240 both of these issues with Algorithm 2, by basing policy selection not each
 241 policy’s weights but on the expected value of each policy at the current
 242 state. As shown in lines 9–12 of Algorithm 3 the value of the greedy action
 243 is found for each base policy, and the policy whose value maximises the dot

244 product with D is set to be active. This altered policy selection is the only
 245 change from Algorithm 2. For ease of reference, from hereon we will refer to
 246 Algorithm 2 as w-steering, as its policy selection is based on each policy’s
 247 weight vector w , and Algorithm 3 as Q-steering, as its policy selection is
 248 based on the $Q_\pi(s, a^*)$ value of each policy.

Algorithm 3 The Q-steering algorithm. Note that the only variation from Algorithm 2 is in the policy selection mechanism (lines 9 to 12).

```

    input: target point  $T$ , set of switching states  $S_S$  which will be equal to  $S_R$ 
    if knowledge exists of recurrent state(s) or equal to  $S$  otherwise, number
    of base policies  $j$ 
1: initialize weight-vectors  $w_1..w_j$  to be evenly distributed within the pos-
    itive quadrant of a unit hypersphere with dimensionality equal to the
    number of objectives
2: initialise state-action values  $Q$  for  $j$  different deterministic policies  $\pi_1.. \pi_j$ 
3: initialise the average reward vector  $R$  to the null vector
4: select at random an active policy  $\pi_A$ 
5: observe initial state  $s$ 
6: while true do
7:   if  $s$  in  $S_S$  then
8:     let  $D = T - R$ 
9:     for each policy  $\pi_i$  do
10:       $Q^*_i = Q_{\pi_i}(s, a^*)$  where  $a^* = \arg \max_a w_i \cdot Q_{\pi_i}(s, a)$ 
11:    end for
12:    select active policy  $\pi_A$  where  $A = \arg \max_{1 \leq A \leq j} D \cdot Q^*_A$ 
13:  end if
14:  execute action based on  $\pi_A$ , observe reward vector  $r$  and next state
     $s'$ 
15:  update record of average reward  $R$ 
16:  update values for  $Q_{\pi_A}$ 
17:  update values for all other  $Q_{\pi_i}$  using off-policy learning
18:   $s \leftarrow s'$ 
19: end while

```

249 Q-steering’s policy selection approach acts as a heuristic to address the
 250 switching-state identification issue. States not visited under a particular
 251 policy π will tend to have lower values for $Q_\pi(s, a^*)$, and so the agent will be
 252 less likely to select π in those states. Instead switching will tend to occur in

the recurrent states as these are visited under all policies. This heuristic also allows the agent to opportunistically switch policies should it find itself in a state which is particularly beneficial with regards to a specific objective [26]. This approach also addresses the policy-identification problem. By basing selection on the actual $Q_\pi(s, a^*)$ values, the policy that is best aligned to move the average reward towards the target will be selected. For example, in the case shown in Figure 4 Q-steering will correctly select policy π_1 .

Q-steering and w-steering share a common implementation issue, regarding the updating of the average reward vector R . Using a cumulative average for R would mean that poor performance on an objective early in learning would influence the value of R and hence affect the agent’s behaviour for an extended period of time. Conversely an exponentially weighted moving average may be biased early in learning by the values used to initialise R . The implementation used in these experiments uses a cumulative average initially, before switching to an exponentially weighted moving average. That is,

$$\mathbf{R}_t = \begin{cases} \sum_{k=0}^t \mathbf{r}_k / t, & \text{if } t \leq L \\ \tau \mathbf{r}_t + (1 - \tau) \mathbf{R}_{t-1}, & \text{otherwise} \end{cases}$$

where $0 < \tau < 1$ and L is a threshold time period. For non-episodic tasks R is updated every time-step and L is defined in time-steps, whereas for episodic tasks R is updated at the end of an episode based on the accumulated rewards over that episode, and L is defined in episodes.

6. Empirical Evaluation of w-Steering and Q-Steering

While [7, 8] provide an extensive theoretical analysis of MDRL, this relies on the assumptions which underpin MDRL and so is not directly applicable to w-steering and Q-steering. Therefore this section empirically compares the two variants of Pareto-optimal steering on three benchmark problems – one in which the set of recurrent states S_R is known, one in which this information is not available to the agent, and one with no recurrent states.

6.1. Benchmark problems

The first benchmark is DST-Mixed (Figure 5), which is a variant of the Deep Sea Treasure task [23]. The agent controls a treasure-hunting submarine, and must compromise between the value of the treasure and the time

Figure 5: (left) The DST-Mixed environment. The agent starts each episode in the top, left cell marked with a submarine. (right) The Pareto front for the DST-Mixed task. Black points show policies which can be found by linear scalarisation (LDS policies).

275 taken to find it. A reward of -1 is given for the time objective on all time-
 276 steps in which a treasure is not found, while the treasure reward is zero except
 277 when a treasure is found at which point it is equal to the treasure value in
 278 the grey cells in Figure 5. The original DST has a concave Pareto front, such
 279 that only two LDS policies exist. As this does not adequately test the ability
 280 of the steering algorithms to select the correct base policy, the DST-Mixed
 281 variant has modified treasure values so that four LDS policies exist. This is
 282 an episodic task with a fixed starting state for all episodes and so that state
 283 can be used as the switching state for steering ($S_R = \{S_{start}\}$).

Figure 6: The Linked Rings benchmark problem.

284 The second benchmark is the non-episodic Linked Rings (Figure 6). Four
 285 Pareto-optimal DS policies exist (see Table 1). Only A and D are LDS poli-
 286 cies, and so these form the base policies for steering. A maximises objective
 287 1 and always moves counter-clockwise, whereas D maximises objective 2 and
 288 always moves clockwise. Table 1 shows that State 1 is the only recurrent

Policy ID	States visited when starting in State 1	Average reward per action
A	1 , 2, 3, 4, 1 , ...	3, -1
B	1 , 2, 1 , ...	1, -0.5
C	1 , 5, 1 , ...	-0.5, 1
D	1 , 5, 6, 7, 1 , ...	-1, 3

Table 1: The four Pareto-optimal DS policies for the Linked Rings.

289 state and so is the correct state in which to switch between base policies,
290 but it is assumed this information is not available to the agent, and so both
291 steering algorithms will treat all states as switching states ($S_S = S$). This
292 benchmark is designed to examine the performance of the two algorithms
293 when no prior knowledge is available about the correct switching states.

Figure 7: The Non-Recurrent Rings benchmark problem.

294 The final benchmark is the non-episodic Non-Recurrent Rings (Figure
295 7). This has a similar structure to the Linked Rings, but the addition of an
296 extra state and arcs means that there is no state which is recurrent under
297 all Pareto-optimal policies (see Table 2). This benchmark tests the ability
298 of the algorithms to operate in an MOMDP without recurrent states. The
299 optimal steering behaviour is to switch between base policies A and D only
300 when in states 1 and 8, but this switching should occur less frequently than
301 in the Linked Rings, as here there is a cost associated with switching policies
302 due to the lower rewards on the arcs between states 1 and 8.

303 6.2. Experimental Method and Metrics

304 The performance of a MORL algorithm may vary depending on where in
305 the Pareto-front the desired solution lies – for example, an algorithm may be
306 good at finding policies which focus on an individual objective, but poor for
307 policies with equal tradeoffs between objectives [9]. For this reason a series

Policy ID	States visited when starting in State 1	Average reward per action
A	1, 2, 3, 4, 1, ...	2, -1
B	1, 2, 1, ...	0.5, -0.5
C	1, 8, 5, 8, 5 ...	-0.5, 0.5
D	1, 8, 5, 6, 7, 8, ...	-1, 2

Table 2: The four Pareto-optimal DS policies for the Non-Recurrent Rings.

of different target points were generated for each of the benchmark problems, distributed at different locations along the convex hull of the Pareto front, thereby ensuring that the algorithms can provide sufficient coverage of the entire front [19]. To satisfy these convex-hull target points, a steering algorithm must be able to identify the correct combination of base policies, as well as the correct ratio with which to combine those policies.

We assume a decision-maker using a steering algorithm will not have knowledge of the exact front and so is unlikely to specify a target point precisely on the front. Therefore it is necessary to test the impact on the algorithm of specifying either a *pessimistic* target (one lying below the front), or an *optimistic* target (one lying above the front). Therefore additional sets of targets were identified by projecting different distances along a line passing through each of the convex-hull target points orthogonal to the slope of the hull. In all cases, if the agent learns the Pareto-optimal policy which minimises the distance to the specified target, then the average reward will be equal to the corresponding convex-hull target point. The target points for each benchmark are listed in Tables A.4, A.5 and A.6 in the Appendix.

For each target, w-steering and Q-steering were trained for 5000 episodes (DST-Mixed) or time-steps (Linked Rings and Non-Recurrent Rings). The ϵ parameter controlling exploration was linearly decayed to zero over this period. Once learning was completed, a further 100 episodes/time-steps were executed in a strictly greedy fashion, and the average reward over this period was recorded. The greedy policy was then evaluated by measuring the Euclidean distance between the average reward and the convex-hull target point matching the target given to the agent. The mean distance over each set of target points of the same degree of pessimism/optimism was then calculated. This process was repeated for ten independent trials, with the mean calculated over these trials. Tabular Q-learning was used as the underlying algorithm for finding base policies for both steering algorithms. Table 3

337 summarises the parameter settings of both algorithms for each benchmark.

Parameter	DST-Mixed	Linked Rings and Non-Recurrent Rings
α (learning rate)	0.9	0.9
λ (eligibility trace decay)	0.95	0.95
initial ϵ	0.9	0.2
γ (discounting)	1.0	0.9
j (number of base policies)	8	4
τ (decay term for averaging)	0.99	0.99
L (time threshold for averaging)	100 episodes	100 timesteps

Table 3: The Q-learning and steering parameters. The number of base policies was set to twice the number of Pareto-optimal LDS policies for each benchmark to ensure that the weight-space sampling identified the required base policies.

338 6.3. Experimental Results and Discussion

339 Figure 8 shows the mean results for both steering algorithms on each of
 340 the benchmark tasks, across the range of optimistic, neutral and pessimistic
 341 targets. The best results theoretically achievable using DS and LDS policies
 342 are also shown. These were determined by selecting the DS or LDS pol-
 343 icy whose average return most closely matches each target. The benefits of
 344 non-stationary policies are evident, with both steering algorithms producing
 345 results which are clearly superior to those achievable using DS or LDS poli-
 346 cies. The only exception is the Non-Recurrent Rings where the absence of
 347 recurrent states causes w-steering to be outperformed by the best DS policies.

348 The behaviour of both steering algorithms is inhibited by the use of pes-
 349 simistic targets, although the results achieved are still superior to LDS and,
 350 in most cases, DS policies. The performance of the steering algorithms is
 351 also quite stable across the range of neutral and optimistic target points,
 352 indicating that they are not overly sensitive to the degree of optimism.

353 On the DST-Mixed benchmark, both steering algorithms were able to
 354 closely match the provided target point. For optimistic targets Q-steering
 355 produced an exact match in most runs, while w-steering was slightly poorer.
 356 For this problem the agents knew the recurrent state, so these differences
 357 must be due to w-steering’s policy identification problem. For the Linked
 358 Rings and Non-Recurrent Rings, policy identification is a less pressing con-
 359 cern as only two unique LDS policies exist. Therefore the improvements
 360 demonstrated by Q-steering on these tasks are due to its ability to handle

Figure 8: Mean distance of average reward from the Pareto hull point corresponding to the target over ten trials of w-steering and Q-steering compared to the best results possible using DS or LDS policies. Error bars show the best and worst result achieved in any trial. For DST-Mixed a zoomed view is also shown.

the switching-state identification problem. On the LinkedRings, Q-steering correctly identified state 1 as the optimal switching state whereas w-steering often switched policies in states 2 and 5, giving rise to sub-optimal returns. Similarly on the Non-Recurrent Rings, w-steering often switched policies in states other than 1 and 8, thereby producing sub-Pareto-optimal results, whereas Q-steering minimised these mistakes.

It should be noted that there was considerably more variation in performance on these tasks compared to DST-Mixed, as shown by the error bars in Figure 8. In particular it was observed that when the target strongly favoured one objective, Q-steering occasionally converged to policies which performed poorly on the less-favoured objective. This occurred in around 10% of runs, and did not occur at all when the target point was balanced between both objectives. This issue appears to be caused by the more fre-

374 quent clearing of eligibility traces, and therefore slower learning, experienced
 375 by the non-active base policies. It may be that higher levels of exploration
 376 are required to ensure that all base policies are learnt correctly under these
 377 circumstances. Nevertheless, even in those runs Q-steering’s performance
 378 was superior to that of the other methods.

379 In summary Q-steering has been shown to learn non-stationary policies
 380 which produce Pareto-optimal results which closely match the specified tar-
 381 get. This represents a substantial improvement over w-steering, addressing
 382 both the switching-state identification and policy-identification problems of
 383 the latter algorithm. Q-steering’s performance is also a major improvement
 384 over the results achievable using stationary deterministic MORL, with reduc-
 385 tions in error with respect to the target of between 40 and 100%. Q-steering’s
 386 performance benefits hold regardless of whether the environment’s recurrent
 387 states are known or not, or even in the case of the Non-Recurrent Rings task,
 388 where there are no recurrent states at all.

389 7. Interactive Q-Steering

390 The results in Section 6.3 demonstrate that Q-steering can produce Pareto-
 391 optimal behaviour when provided with a suitable choice of target point, par-
 392 ticularly if that target is slightly optimistic with regards to the Pareto front.
 393 For many problems, it should be straightforward for a human decision-maker
 394 to specify such a target to indicate their desired preference between objec-
 395 tives. For example, if MORL is being applied to a task where prior measures
 396 of performance of alternative methods such as human control exist, then
 397 those measures can be used as a guide when specifying a target.

Figure 9: An abstract view of interactive Q-steering. The visualisation of the front is updated as the agent learns more accurate estimates of the value of each base policy, and is used to inform the user’s choice of target point.

398 However in some cases there may be little information available to guide
 399 the user in specifying a target. It is also possible that the preferred trade-offs

may vary over time as circumstances not captured in the environmental state change (for example, a coach of a Robocup team may instruct their agents to focus more on a defensive objective and less on scoring when defending a narrow lead late in a game). Q-steering can be applied to these scenarios by allowing the decision-maker to interactively change the target during or after learning. As the Q-steering agent has learnt base policies covering the entire Pareto hull, when the target changes it should be able to switch its choice of active base policies so as to steer towards the new target. To aid the user in making an informed choice of target, the agent should maintain an estimate of the Pareto-front and provide a visualisation of this to the user, who may then alter the specification of the target, as shown in Figure 9.

Interaction between a human user and an agent in the context of MORL has previously been explored by several authors. Most commonly this has been in the context of multi-policy learning for decision support, where the agent’s role is to identify the possible range of behaviours available, with the final choice of trade-offs between objectives then made by the human [19, 5, 27]. In this scenario the interaction between the human and agent takes place only once learning has been completed, in contrast to interactive Q-learning where the human decision-maker can be involved throughout the learning process. This form of ongoing interaction has also previously been considered, most notably in work on preference-based RL (PBRL) [28, 29]. A PBRL agent learns in the absence of a numerical reward signal, on the basis of qualitative feedback provided by the human user in the form of preferences over either actions, trajectories or policies. As such it depends on continued interaction with the user throughout the learning process. [29] comment on the similarities which exist between PBRL and MORL – however, they also note that MORL can be seen as a special case of the PBRL framework in that a numerical reward signal is available, and that therefore algorithms which make use of that reward information (such as Q-steering) are likely to be more effective than general PBRL algorithms.

7.1. Visualising the Estimated Pareto Front

To visualise the front, an estimate is required of the average-reward value P_{π_i} for each base policy π_i . These can then be displayed to the user in either graphical or tabular format. For problems with a single known recurrent state (such as DST-Mixed), P_{π_i} can be approximated by the value of the greedy action for that state for that policy. More generally if S_R contains multiple states (as in an episodic task with multiple starting states), then

437 P_{π_i} must be an average of the greedy action values for the policy across those
 438 states, weighted by the frequency with which those states are visited.

439 Where the recurrent states are unknown, the estimation of the policy
 440 value is more complicated. Averaging the value over all states would likely
 441 underestimate the true value, as it would include states which are not encoun-
 442 tered while following that policy. Therefore we restrict the calculations to
 443 only update the value of the current active policy π_A at each state. A further
 444 complication arises if the task involves discounting, as the Q-values learnt
 445 by the agent correspond to the expected discounted sum of future rewards
 446 whereas the target is specified in terms of average reward, as this is more
 447 convenient for the human decision-maker. An estimate of the average-reward
 448 value of a policy can be derived by multiplying the Q-value by $1 - \gamma$.

Algorithm 4 Deriving an estimate of the Pareto front from base policy Q-values

input: set of switching states S_S , set of known recurrent states S_R (pos-
 sibly empty), number of base policies j , state s , $Q_{\pi_i}(s, a)$ for each base
 policy, active base policy π_A , discounting rate γ , time threshold L

```

1: if  $s$  in  $S_S$  then
2:   for each policy  $\pi_i$  do
3:     if  $(\pi_i = \pi_A)$  or  $(s$  in  $S_R)$  then
4:        $Q^*_i = Q_{\pi_i}(s, a^*)$  where  $a^* = \arg \max_a Q_{\pi_i}(s, a)$ 
5:       if  $t \leq L$  then
6:          $Q_{\pi_i, t} = \sum_{k=0}^t Q^*_i / t$ 
7:       else
8:          $Q_{\pi_i, t} = \tau Q^*_i + (1 - \tau) Q_{\pi_i, t-1}$ 
9:       end if
10:      if  $\gamma = 1$  then
11:         $P_{\pi_i} = Q_{\pi_i}$ 
12:      else
13:         $P_{\pi_i} = Q_{\pi_i}(1 - \gamma)$ 
14:      end if
15:    end if
16:  end for
17: end if

```

449 Algorithm 4 details the complete process used to estimate the Pareto
 450 front points from the Q-values of each base policy. Note that t is defined in

451 episodes for episodic tasks, and in time-steps for non-episodic tasks.

452 7.2. Empirical Evaluation of Interactive Q-steering

Figure 10: The estimated Pareto front produced at various points in learning for a randomly-selected run on each of the benchmarks.

453 To simulate the user interaction with Q-steering as described in the pre-
 454 vious subsections, the following experimental methodology was applied:

- 455 • The target T was initialised to an overly optimistic target T_I , based on the
 456 maximum value obtainable for each objective for that benchmark.
- 457 • Q-steering was executed with estimates of the Pareto front derived at reg-
 458 ular intervals during learning using Algorithm 4.
- 459 • To simulate the user changing the target, after 2000 episodes or timesteps
 460 T was changed to a final target point T_F , and learning continued. The
 461 value of T_F was selected from the set of target points at distance 1 from
 462 the Pareto hull of the benchmark, as listed in Tables A.4, A.5 and A.6.

- This process was repeated 10 times for each target for each benchmark.
The performance was measured in terms of the distance between the average reward R and the Pareto hull point corresponding to target T_F .

Figure 11: Mean distance of average reward from the Pareto hull point corresponding to the final target T_F over ten trials for Q-steering with $T = T_F$ throughout training, and for interactive Q-steering where T switches from T_I to T_F after 2000 episodes or time-steps. The dashed vertical line marks the stage at which the target was changed.

As shown in Figure 10, the initial estimate of the front for each benchmark is poor, but improves rapidly as the agent learns. In all cases after 1000 time-steps or episodes the visualisation of the front is sufficiently accurate to allow the user to modify the target. Figure 11 shows that early in learning, the error with respect to the Pareto hull point corresponding to T_F is quite high – this is expected as during this period the agent is trying to optimise its

behaviour with regards to the initial target T_I . Once a suitable visualisation of the front is obtained and the target point T changed to T_F there is a sharp drop in error, as the agent immediately modifies its policy selection to satisfy this new target. In fact on the Linked Rings and Non-Recurrent Rings, the performance of the interactive agent with respect to T_F is surprisingly slightly improved over that of a Q-steering agent using $T = T_F$ throughout training. This was due to the interactive agent avoiding the occasional sub-optimal runs of Q-steering when using a target which favours one objective, as reported in Section 6.3. This indicates that there may be benefit in initially training on a target which provides a balanced trade-off between objectives, even if the eventual desired policy should favour one of the objectives.

8. A Practical Demonstration of Q-Steering: Battery Control

While Section 6 used tabular Q-learning, Q-steering is also applicable to problems where the state space is too large for tabular methods to be appropriate. To illustrate this, this section reports results obtained by applying Q-steering in conjunction with tile-coding to a simulation of controlling local battery storage for a residential photovoltaic power generation system.

Local storage allows the power generated to be ‘time-shifted’ to better match the household’s demand profile, delivering benefits in energy purchase costs and environmental impact. A simple solution is to over-size the battery, but this has a significant capital cost – a typical long-duration battery can cost between AUS\$400/kWh and AUS\$1000/kWh [30]. As such, the first objective is to reduce the amount of external power consumed by the household for a small battery system. An ideal storage system will dynamically match load through rapid charge and discharge response. However while many battery chemistries can supply and sink a significant amount of power for short periods, this adversely impacts on their energy capacity and lifespan, particularly when done repeatedly over long periods of time [31]. So a second, conflicting objective is to minimise the wear on the battery.

The simulation is based on a simplified battery model with linear dynamics, assuming a round-trip efficiency of 90% (with a fixed 10% loss during charging). It is also assumed that the long-term battery wear can be approximated by the amount discharged or absorbed by the battery at each time step. The power generated by the solar panels and the load consumed are derived from one year’s worth of historical data from a real household. The agent acts on an hourly basis, selecting between 4 different levels of charging

508 or discharging the battery, or to not use the battery at all, for a total of
509 9 actions. The state information consists of five continuous variables (the
510 battery charge level and perfect forecasts of load and generation for both one
511 and six hours ahead), so tabular methods are impractical. Instead the agent
512 uses tile-coding [32] to represent the values for each base policy.

Figure 12: The approximate Pareto front found by scalarised Q-learning on the battery control task, and the results found by Q-steering using two different optimistic target points. Both objectives are measured in terms of the mean hourly value in Wh.

513 Figure 12 shows the final approximate Pareto front discovered by multiple
514 runs of scalarised Q-learning (which can only find LDS policies) and Q-
515 steering. For scalarised Q-learning the objective weights tested were evenly
516 spaced at increments of 0.1 in the range 0..1. Some runs produced results
517 which were Pareto-dominated, or very similar to those from other runs so
518 for clarity these have been omitted from the figure. It was observed that
519 even small increases in the weights assigned to the battery wear objective
520 resulted in scalarised Q-learning converging to policies that were strongly
521 biased towards this objective, making very little use of the battery. Therefore
522 an approximation of the Pareto front using uniformly spaced sets of weights
523 failed to find any policies which provided equitable trade-offs between the
524 objectives. In contrast when target points representing such trade-offs (-
525 15, -135) and (-20, -134) were provided to Q-steering, the policies found
526 corresponded quite closely to the specified targets (note that the target points
527 were chosen optimistically and can not actually be achieved). The results
528 achieved by Q-steering are also comparable to those produced by several

529 hand-crafted policies derived from expert knowledge. Therefore the use of
 530 non-stationary policies as learnt by Q-steering has enabled a closer match
 531 to the user’s desired outcomes than could be achieved using deterministic
 532 stationary MORL methods.

533 9. Conclusion and Future Work

534 Learning Pareto-optimal non-stationary policies is important for many
 535 MORL tasks, as these policies may be superior (in terms of Pareto-dominance)
 536 to deterministic stationary policies, and better able to match the preferred
 537 trade-off between objectives. The MDRL steering algorithm [7, 8] forms a
 538 non-stationary mixture of multiple deterministic base policies conditioned on
 539 the vector of rewards received so far. This paper has examined two adapta-
 540 tions of steering (w-steering and Q-steering) designed for learning a Pareto-
 541 optimal policy, with the preference between objectives specified via a target
 542 point in reward space. w-steering is a direct adaptation of MDRL, while Q-
 543 steering incorporates a novel policy-selection heuristic designed to improve its
 544 performance, particularly for environments where the agent cannot be pro-
 545 vided with prior knowledge about the recurrent states of the environment.

546 The empirical results presented show that both Pareto-optimal steering al-
 547 gorithms exploit the benefits of non-stationarity to substantially outperform
 548 deterministic stationary policies, producing a reduction in error on the order
 549 of 40-100% on the benchmark problems. In addition by taking into account
 550 the current state of the environment Q-steering substantially outperforms
 551 w-steering where knowledge of the recurrent states is not available, or where
 552 there are no recurrent states. This represents a significant improvement over
 553 the original formulation of the steering approach to MORL, offering scope
 554 for application to a much broader class of problems. In addition the re-
 555 sults reported on the episodic DST-Mixed task indicate that Q-steering also
 556 addresses the policy-identification problem of w-steering, producing slightly
 557 improved results for optimistic targets even when the recurrent state infor-
 558 mation is available.

559 Q-steering also supports interactive exploration of a multiobjective task
 560 by a human user. Section 7 demonstrated that the Pareto front can be
 561 estimated while Q-steering learns from an optimistic target point. This es-
 562 timated front allows the user to make a more informed specification of the
 563 target, which produces an immediate change in the behaviour of the agent.

564 Therefore Q-steering can be utilised by a decision-maker even with little prior
565 knowledge of the outcomes achievable for each objective.

566 The ability of Q-steering to scale to more complex problems has also been
567 demonstrated on a simulated battery control task. The state space of this
568 task has sufficient dimensions to render tabular methods impractical, and so
569 a form of function approximation is required to store the Q-values (in this
570 case, tile-coding was used). Function approximation can be incorporated
571 into Q-steering with no further alteration to the algorithm, as demonstrated
572 by the results observed on this task, where Q-steering was able to match the
573 specified targets much more closely than scalarised Q-learning.

574 While Q-steering represents a significant addition to the arsenal of MORL
575 algorithms, a number of issues still require further investigation:

- 576 • In Section 6.3 it was observed that Q-steering failed to converge to a fully
577 optimal policy in a small percentage of runs where the target point strongly
578 favoured one objective. The results in Section 7 indicated this did not oc-
579 cur when the target was balanced with regards to both objectives, which
580 suggests the cause is that an unbalanced target leads to some base policies
581 being active only infrequently, resulting in their learning being slowed due
582 to more frequent clearing of the eligibility traces. Future work should in-
583 vestigate whether alternative approaches to exploration (such as optimistic
584 initialisation, or exploratory selection at policy rather than action level)
585 may ameliorate this issue [33].
- 586 • During the experiments reported in Section 8 issues were encountered with
587 the memory requirements of Q-steering – even with the use of tile-coding,
588 the need to learn Q-values for many base policies leads to high memory
589 usage compared to single-policy methods. As noted in [12] the optimal
590 steering policy for a problem with n objectives will only use a maximum
591 of n base policies. However the current formulation of Q-steering requires
592 that policies be learnt for many different weight sets in order to ensure
593 that the required base policies are discovered. Methods for minimising the
594 number of base policies, such as by adapting the weights during learning
595 [25], would be beneficial in reducing the memory overhead of Q-steering.
- 596 • The results reported in this paper use Q-learning as the algorithm for
597 base policies. However previous research has reported divergence issues
598 for Q-learning in conjunction with function-approximation [34]. While

such problems were not observed during the experiments in Section 8, if Q-steering is to be effectively applied to large problems it will be necessary to use more stable off-policy learning methods as described in [35]. However this is merely an implementation issue as any off-policy temporal-difference algorithm can readily be incorporated into the Q-steering algorithm.

It should also be noted that non-stationary policies may be inappropriate in certain applications. The patient treatment scenario examined by [36] is an excellent example. A non-stationary policy which switches between one base policy which minimises symptoms but also maximises side effects and another which does the reverse may appear to produce an excellent trade-off between those objectives when rewards are averaged over multiple episodes of treating different patients. However the experience of any individual patient is unlikely to be desirable. Therefore there still exists a need to develop better algorithms for finding stationary deterministic policies for such tasks. The recent Pareto-set algorithm [37, 38] represents a promising new approach for identifying such policies.

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Appendix A. Target points used in steering experiments

Distance from hull	DST-Mixed
-1	(5.0939,-3.9138), (15.223,-11.9866), (18.2922,-16.7873)
-0.5	(5.2969,-3.4569), (15.5865,-11.6433), (18.7711,-16.6437)
0	(5.5,-3), (15.95,-11.3), (19.25,-16.5)
0.5	(5.7031,-2.5431), (16.3135,-10.9567), (19.7289,-16.3563)
1	(5.9061,-2.0862), (16.677,-10.6134), (20.2078,-16.2127)
1.5	(6.1092,-1.6293), (17.0405,-10.2701), (20.6867,-16.069)
2	(6.3123,-1.1724), (17.404,-9.9268), (21.1657,-15.9253)

Table A.4: The target points used for DST-Mixed. Negative distances indicate pessimistic targets, 0 indicates neutral targets, and positive distances correspond to optimistic targets. Values have been rounded to 4 decimal places for purposes of display.

Distance from hull	Linked Rings
-1	(-0.7071,1.2929), (0.2929,0.2929), (1.2929,-0.7071)
-0.5	(-0.3536,1.6464), (0.6464,0.6464), (1.6464,-0.3536)
0	(0,2), (1,1), (2,0)
0.5	(0.3536,2.3536), (1.3536,1.3536), (2.3536,0.3536)
1	(0.7071,2.7071), (1.7071,1.7071), (2.7071,0.7071)
1.5	(1.0607,3.0607), (2.0607,2.0607), (3.0607,1.0607)
2	(1.4142,3.4142), (2.4142,2.4142), (3.4142,1.4142)

Table A.5: The target points used for Linked Rings

Distance from hull	Non-Recurrent Rings
-1	(-0.9571,0.5429), (-0.2071,-0.2071), (0.5429,-0.9571)
-0.5	(-0.6036,0.8964), (0.1464,0.1464), (0.8964,-0.6036)
0	(-0.25,1.25), (0.5,0.5), (1.25,-0.25)
0.5	(0.1036,1.6036), (0.8536,0.8536), (1.6036,0.1036)
1	(0.4571,1.9571), (1.2071,1.2071), (1.9571,0.4571)
1.5	(0.8107,2.3107), (1.5607,1.5607), (2.3107,0.8107)
2	(1.1642,2.6642), (1.9142,1.9142), (2.6642,1.1642)

Table A.6: The target points used for Non-Recurrent Rings

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