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Softmax Exploration Strategies for Multiobjective Reinforcement Learning

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Abstract

Despite growing interest over recent years in applying reinforcement learning to multiobjective problems, there has been little research into the applicability and effectiveness of exploration strategies within the multiobjective context. This work considers several widely-used approaches to exploration from the single-objective reinforcement learning literature, and examines their incorporation into multiobjective Q-learning. In particular this paper proposes two novel approaches which extend the softmax operator to work with vector-valued rewards. The performance of these exploration strategies is evaluated across a set of benchmark environments. Issues arising from the multiobjective formulation of these benchmarks which impact on the performance of the exploration strategies are identified. It is shown that of the techniques considered, the combination of the novel softmax-epsilon exploration with optimistic initialisation provides the most effective trade-off between exploration and exploitation.

Keywords: multiobjective reinforcement learning, exploration, ϵ -greedy exploration, optimistic initialisation, softmax

1. Introduction

Most reinforcement learning (RL) algorithms consider only a single objective, encoded in a scalar reward. However, many sequential decision-making tasks naturally have multiple conflicting objectives, such as minimising both distance and risk of congestion in path-finding [1, 2], optimizing both energy efficiency and quality of communication in wireless networks [3], and control tasks with multiple performance indices [4]. Therefore recent years have seen

8 multiobjective reinforcement learning (MORL) emerge as a growing area of
 9 research [5]. One issue which has yet to be investigated to any significant
 10 extent is the role of exploration in a multiobjective context. Balancing the
 11 exploitation of the agent’s current learning against the potential to improve
 12 the current policy through exploratory actions is critical to the performance
 13 of a RL agent, particularly in online learning [6]. While there has recently
 14 been extensive examination of this trade-off for multiobjective multi-armed
 15 bandits (e.g. [7, 8]), there has been no work yet addressing exploration in
 16 general multiobjective environments with multiple states.

17 This paper considers three approaches to exploration which have been
 18 widely used in the single-objective reinforcement learning literature (ϵ -greedy
 19 exploration, softmax exploration and optimistic initialisation), and examines
 20 how they can be applied in the context of multiobjective reinforcement learn-
 21 ing. The three methods are incorporated into a multiobjective formulation
 22 of the $Q(\lambda)$ learning algorithm, which in some cases requires modifications to
 23 aspects of the exploration algorithm, and then evaluated across three bench-
 24 mark multiobjective environments. The results of these empirical evaluations
 25 demonstrate that exploration in a multiobjective environment differs from ex-
 26 ploration in single-objective reinforcement learning, and provide insight into
 27 the correct choice of exploration strategy and settings.

28 2. Background

29 This section of the paper provides the necessary background informa-
 30 tion on multiobjective reinforcement learning (and specifically value-based
 31 approaches to MORL), and single-objective exploration methods. Section
 32 3 will build on this background to explore what modifications, if any, are
 33 required to adapt these exploration methods to apply to multiobjective RL.

34 2.1. Multiobjective Reinforcement Learning

35 The fundamental difference between single-objective and multiobjective
 36 RL is that the latter has vector-valued rewards (with each element of the
 37 vector corresponding to one of the objectives) rather than scalar rewards.
 38 Value-based approaches to RL can therefore be extended to MORL by simply
 39 storing a vector value for each state-action pair. That is, instead of the value
 40 of an action a in a state s being presented by a scalar $Q(s, a)$, it is represented
 41 by the vector value $\bar{Q}(s, a)$. For example Algorithm 1 outlines how Q-learning
 42 can be extended to handle multiple objectives. Note that in this algorithm

43 and for the remainder of the paper, $Q(s, a, o)$ denotes the scalar value of
 44 action a in state s with respect to objective o , $\bar{Q}(s, a)$ denotes the vector
 45 value of action a in state s for all objectives, and $\bar{Q}(s)$ denotes the vector
 46 values of all actions in state s .

47 The primary issue is then how to perform action selection to form a
 48 policy on the basis of these vector values, as required by lines 9, 12 and
 49 13 of Algorithm 1. In single-objective RL, one or more actions will have a
 50 $Q(s, a)$ value which exceeds or equals the value of all other actions, and so the
 51 greedy policy simply selects one of these actions. However when the $Q(s, a)$
 52 values are vectors action selection is no longer as simple, as one action may
 53 be maximal for one objective, and a different action be optimal with regards
 54 to another objective. Formally, there may be multiple actions which are
 55 non-dominated in terms of the Pareto-dominance relationship [5]. Therefore
 56 the agent must incorporate an action-selection function which defines an
 57 ordering over these vector values, so as to allow the greedy action to be
 58 identified as shown in Equation 1. U has been used to represent this action-
 59 selection function in reference to the concept of *utility* which forms the basis
 60 for economic theories of preference [9].

$$U(\bar{Q}(s, a), \bar{Q}(s, b)) = \begin{cases} +1, & \text{if } a \text{ is preferred to } b \ (\bar{Q}(s, a) \succ \bar{Q}(s, b)) \\ -1, & \text{if } b \text{ is preferred to } a \ (\bar{Q}(s, a) \prec \bar{Q}(s, b)) \\ 0, & \text{otherwise } (\bar{Q}(s, a) \sim \bar{Q}(s, b)) \end{cases} \quad (1)$$

61 Commonly action selection is implemented via a scalarisation function f
 62 which maps the vectors to scalar values [10], as shown in Equation 2.

$$U_{\text{scalar}}(\bar{Q}(s, a), \bar{Q}(s, b)) = \begin{cases} +1, & \text{if } f(\bar{Q}(s, a)) > f(\bar{Q}(s, b)) \\ -1, & \text{if } f(\bar{Q}(s, a)) < f(\bar{Q}(s, b)) \\ 0, & \text{if } f(\bar{Q}(s, a)) = f(\bar{Q}(s, b)) \end{cases} \quad (2)$$

63 The scalarisation function is often parameterised so that the desired
 64 trade-offs between objectives can be expressed via these parameters [5]. One
 65 approach to scalarisation is to calculate a linear weighted sum of the objec-
 66 tive values as shown in Equation 3, where the weights w_o indicate the relative
 67 importance of the objectives:

$$f(\bar{Q}(s, a)) = \sum_{o=1}^n w_o Q(s, a, o) \quad (3)$$

Algorithm 1 Multiobjective $Q(\lambda)$

input: learning rate α , discounting term γ , eligibility trace decay term λ , number of objectives n

- 1: **for** all states s , actions a and objectives o **do**
- 2: initialise $Q(s, a, o)$ $\triangleright$ e.g. to 0, or to optimistic values
- 3: **end for**
- 4: **for** each episode **do**
- 5: **for** all states s and actions a **do**
- 6: $e(s, a) = 0$
- 7: **end for**
- 8: observe initial state s_t
- 9: select action a_t based on an exploratory policy derived from $\bar{Q}(s_t)$
- 10: **for** each step of the episode **do**
- 11: execute action a_t , observe s' and reward vector $\bar{r}$
- 12: select action a^* based on a greedy policy derived from $\bar{Q}(s')$
- 13: select action a' based on an exploratory policy derived from $\bar{Q}(s')$
- 14: **for** each objective o **do**
- 15: $\delta_o = r_o + \gamma Q(s', a^*, o) - Q(s_t, a_t, o)$
- 16: **end for**
- 17: set $e(s_t, a_t) = 1$
- 18: **for** each state s and action a **do**
- 19: **for** each objective o **do**
- 20: set $Q(s, a, o) = Q(s, a, o) + \alpha \delta_o e(s, a)$
- 21: **end for**
- 22: **if** $a' = a^*$ **then**
- 23: set $e(s, a) = \gamma \lambda e(s, a)$
- 24: **else**
- 25: set $e(s, a) = 0$
- 26: **end if**
- 27: **end for**
- 28: $s_t = s', a_t = a'$
- 29: **end for**
- 30: **end for**

Linear scalarisation is simple to implement and remains the most widely used form of MORL, particularly in applied research (e.g. [11, 12]). However there are contexts in which it can not adequately express the desired trade-off between objectives [5] or discover the most appropriate policy [13].

Consequently several researchers have examined the use of non-linear approaches to action selection, such as thresholded lexicographic ordering [14], the hypervolume metric [15], and Chebyshev distance [16]. These non-linear approaches can be a more appropriate means of specifying the required trade-offs between objectives for many problems. For example, thresholded lexicographic ordering is a natural means of specifying the desire to maximise one objective subject to achieving threshold levels of performance on the other objective(s).

As noted by [5], linear scalarisation of the reward vector essentially converts the problem from a multiobjective Markov Decision Process (MOMDP) into a more conventional single-objective MDP. Therefore exploration methods will function in the same manner as they do for single-objective RL. Hence the more interesting case is the application of exploration methods to MORL in conjunction with non-linear approaches to action selection. For the purposes of this study we will focus on the thresholded lexicographic ordering (TLO) approach to action-selection as this is the most widely used non-linear approach [14, 17, 18, 19]. However the methods developed and tested within this study are equally applicable to other forms of non-linear action selection.

The foundation of TLO is a lexicographic ordering of vectors as defined in Algorithm 2. TLO simply applies a thresholding operation to each element of the vectors $\bar{A}$ and $\bar{B}$ prior to performing lexicographic comparison, as shown in Algorithm 3. Hence the aim of an agent using TLO-based action selection is to maximise the value of objective n , subject to satisfying the constraints of achieving threshold values for the other objectives. By comparing the Q-values of all pairs of actions available in the current state, an agent can identify the greedy action to select. In many problems the threshold values arise from a definition of the problem and the desired outcome – for example, an agent controlling a self-driving vehicle may aim to minimise travel time, subject to keeping the risk of an accident below an acceptable level. Alternatively, as will be the case in the experiments reported in this paper, an approximation to the complete Pareto front of policies might be achieved by repeatedly executing a single-policy learning algorithm based on TLO with different sets of threshold values.

Algorithm 2 $U_{lexicographic}$: Action selection based on lexicographic ordering

input: $\bar{A}$ and $\bar{B}$ (two vectors of length n)
output: returns +1 if $\bar{A}$ is lexicographically superior to $\bar{B}$, -1 if $\bar{B}$ is superior, and 0 if $\bar{A}$ and $\bar{B}$ are equally preferred

```
1: for each  $i$  in  $1..n$  do
2:   if  $A_i > B_i$  then
3:     return +1
4:   else
5:     if  $B_i > A_i$  then
6:       return -1
7:     end if
8:   end if
9: end for
10: return 0
```

Algorithm 3 U_{TLO} : Action selection based on Thresholded Lexicographic Ordering (TLO)

input: $\bar{A}$ and $\bar{B}$ (two vectors of length n), a vector of thresholds $\bar{Th}$ of length $n - 1$
output: returns +1 if $\bar{A}$ is TLO superior to $\bar{B}$, -1 if $\bar{B}$ is superior, and 0 if $\bar{A}$ and $\bar{B}$ are equally preferred

```
1: for each  $i$  in  $1..n - 1$  do
2:   set  $T_i^A = \min(A_i, Th_i)$ 
3:   set  $T_i^B = \min(B_i, Th_i)$ 
4: end for
5: set  $lex = U_{lexicographic}(\bar{T}^A, \bar{T}^B)$  ▷ see Algorithm 2
6: if  $lex \neq 0$  then
7:   return  $lex$ 
8: else
9:   return  $U_{lexicographic}(\bar{A}, \bar{B})$  ▷ see Algorithm 2
10: end if
```

106 2.2. Exploration in Single-Objective RL

107 As noted in the introduction, the tradeoff between exploration and ex-
 108 ploitation is a critical factor in the performance of a RL agent. Value-based
 109 RL methods require that all states and actions are experienced sufficiently
 110 often to allow the true value function to be learnt. An insufficient level of
 111 exploration may lead to learning of a sub-optimal policy. However over-
 112 exploration can lead to poor performance during learning, which may be
 113 unacceptable, particularly when learning in an online situation. While this
 114 exploration-exploitation tradeoff in single-objective RL has been widely stud-
 115 ied (for example, [20, 21, 22]) in practice three relatively simple approaches
 116 remain widely used. These are ϵ -greedy exploration, softmax exploration and
 117 optimistic initialisation.

118 An ϵ -greedy strategy, at each time step, either selects the greedy action
 119 with probability $1-\epsilon$, or selects an action at random with probability ϵ . The
 120 value of ϵ is generally decayed over time to 0 so that the agent eventually
 121 converges to following a strictly greedy policy. The exploitation-exploration
 122 trade-off is controlled via the initial value of ϵ , with higher values resulting in
 123 a greater emphasis on exploration. The primary limitation of this approach
 124 is its randomness – all non-greedy actions are treated equally regardless of
 125 how likely it is that they might in fact yield an improvement in policy.

126 Softmax exploration provides a somewhat more sophisticated approach
 127 by taking into account the relative values of all actions. The probability of
 128 selection of each action is determined by Equation 4.

$$P(a) = \frac{e^{Q(s,a)/\tau}}{\sum_{b=1}^n e^{Q(s,b)/\tau}} \quad (4)$$

129 As with ϵ -greedy exploration, the extent to which exploration takes place
 130 is controlled by a single parameter – in this case the temperature or τ pa-
 131 rameter. At high temperatures all actions will be selected with almost equal
 132 probability regardless of their estimated value. The temperature is generally
 133 reduced during learning, leading the agent to increasingly focus on higher-
 134 valued actions. As the temperature approaches zero, the agent’s choice of
 135 action converges towards a strictly greedy policy. The primary criticism of
 136 softmax is that the setting of the temperature parameter is unintuitive, par-
 137 ticularly in comparison to the straightforward interpretation of ϵ in ϵ -greedy
 138 exploration.

139 Another commonly-used approach to producing more directed exploration

140 is via optimistic initialisation of state-action values. By setting the initial val-
 141 ues to be higher than their expected final values, actions which have been
 142 relatively infrequently executed will tend to be valued more highly than other
 143 actions in the same state. In this way the agent is encouraged to perform
 144 a broad exploration of the states and actions of the environment even when
 145 selecting actions greedily based on its current Q-values [23].

146 3. Exploration in Multiobjective RL

147 In this section we will consider how each of the exploration strategies
 148 discussed in 2.2 can be applied in the context of multiple objectives.

149 Extending ϵ -greedy exploration to multiple objectives is straightforward.
 150 When an action is to be selected greedily, then the appropriate multi-objective
 151 action selection operation is used (TLO in our case). Otherwise an action
 152 is selected randomly. This has been the predominant exploration approach
 153 adopted in the MORL literature so far [12, 15, 16, 19, 24, 25, 26, 27, 28, 29,
 154 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45].

155 Similarly optimistic initialisation can be applied quite easily to the multi-
 156 objective context. The only consideration is that as the range of values may
 157 vary between the different objectives, the optimal choice of initial optimistic
 158 value may also differ. Therefore a vector will be used to define the initial
 159 values, so that each objective may have its own optimistic initialisation.

160 Extending softmax to handle multiple objectives is more complicated, as
 161 Equation 4 can not be directly applied if $Q(s, a)$ is not a scalar. The appro-
 162 priate approach to modifying softmax to handle vector Q values depends on
 163 the nature of the multiobjective action selection method being used. If the
 164 action selection operation is based on a scalarisation operator which maps
 165 the vector value to a scalar, then the Q values can simply be scalarised
 166 prior to applying the softmax equation. For example this approach could be
 167 used if the multiobjective action selection is being implemented via linear
 168 weighted scalarisation. This approach is shown in Equation 5, where $P(a)$ is
 169 the probability of selecting action a , f is the scalarisation function, and n is
 170 the number of actions.

$$P(a) = \frac{e^{f(\bar{Q}(s,a))/\tau}}{\sum_{b=1}^n e^{f(\bar{Q}(s,b))/\tau}} \quad (5)$$

171 However some approaches to multiobjective action selection can not read-
 172 ily be phrased as an explicit scalarisation operation. For example, the TLO

173 approach can not be described by a simple scalarisation, and therefore Equa-
 174 tion 5 can not be applied. Instead we need to define another means of
 175 converting the Q-value vectors into scalar values to which the softmax op-
 176 erator can be applied. This process should be consistent with the order-
 177 ing of actions under the action selection operation – that is, if action a is
 178 ranked higher than action b by the non-linear action selection operator (i.e.
 179 if $U(\bar{Q}(s, a), \bar{Q}(s, b)) > 0$), it should be more likely to be selected than b when
 180 the softmax operator is applied.

181 In this paper we propose two methods of achieving this aim. The first
 182 approach, which we will refer to as *softmax-tournament* or *softmax-t*, is based
 183 upon ranking the actions using the action-selection operator. Each action is
 184 assigned a score equal to the number of other actions which it is ranked
 185 above or equal with. These scores are then used as the basis for the softmax
 186 selection, as described in Algorithm 4.

Algorithm 4 Softmax-tournament selection based on a non-linear action-
 selection operator

input: $\bar{Q}(s)$ for the current state s , non-linear action comparison function
 U (for example, U_{TLO} as defined in Algorithm 3)
 output: P_a defining the probability of selection for each action a

```

1: for each  $a$  do
2:   set  $u_a = 0$ 
3:   for all other actions  $b$  do
4:     if  $U(\bar{Q}(s, a), \bar{Q}(s, b)) \geq 0$  then
5:        $u_a = u_a + 1$ 
6:     end if
7:   end for
8: end for
9: for each action  $a$  do
10:  set  $P_a = \frac{e^{u_a/\tau}}{\sum_{b=1}^n e^{u_b/\tau}}$ 
11: end for
```

187 The softmax-t approach ensures that the probability with which an action
 188 will be selected is related to its ranking by the action selection operator –
 189 in this sense it is *consistent* with the action selection operator. However
 190 the use of tournament scores may obscure the magnitude of the differences
 191 between actions. For example the highest ranked action will be selected

192 with the same probability if it is only valued slightly higher than the second-
 193 ranked action, or if it is valued substantially higher than the second-ranked
 194 action. The second approach we propose addresses this by basing the score
 195 of each action on the additive-epsilon metric (ϵ_+). This has been widely used
 196 in evolutionary multiobjective optimisation [46], and has previously been
 197 suggested as a metric for evaluating MORL algorithms by [47]. The additive-
 198 epsilon metric for an action is calculated as the maximum distance by which
 199 that action is valued below the greedy action on any individual objective, as
 200 shown in Figure 1. To make this value suitable for softmax selection, it needs
 201 to be inverted (so that a greater additive epsilon value results in a lower
 202 score). In addition the additive epsilon values should be calculated based
 203 on normalised objective values, to prevent an objective with a larger range
 204 from dominating these calculations. The overall *softmax-epsilon* (*softmax-E*)
 205 approach is summarised in Algorithm 5.

Figure 1: An illustration of the additive-epsilon metric. a_G indicates the value of the greedy action with respect to each objective. a_A and a_B show the value of two other actions, and ϵ_A and ϵ_B indicate the additive-epsilon error metric for each of those actions.

206 4. Experimental Methodology

207 The remainder of this paper presents an empirical comparison of the
 208 multiobjective exploration strategies described in Section 3. To evaluate the
 209 effectiveness of each method in balancing the trade-off between exploration
 210 and exploitation it is necessary to consider the agent’s performance both
 211 during learning (its online performance) and also the quality of the final
 212 greedy policy learnt by the agent (its offline performance). Ideally an agent
 213 should perform well on both of these measures.

Algorithm 5 Softmax-epsilon selection based on a non-linear action-selection operator

input: $\bar{Q}(s)$ for the current state s , non-linear action comparison function U (for example, U_{TLO} as defined in Algorithm 3)
output: P_a defining the probability of selection for each action a

- 1: **for** each objective o **do**
- 2: set min_o = minimum value of $Q(s, a, o)$ across all actions a
- 3: set max_o = maximum value of $Q(s, a, o)$ across all actions a
- 4: **end for**
- 5: set g to be the greedy action, as defined by U
- 6: **for** each action a **do**
- 7: set $u_a = 0$
- 8: **for** each objective o **do**
- 9: set $u_\epsilon = \frac{Q(s, g, o) - min_o}{max_o - min_o} - \frac{Q(s, a, o) - min_o}{max_o - min_o}$
- 10: **if** $u_\epsilon > u_a$ **then**
- 11: $u_a = u_\epsilon$
- 12: **end if**
- 13: $u_a = 1 - u_a$
- 14: **end for**
- 15: **end for**
- 16: **for** each action a **do**
- 17: set $P_a = \frac{e^{u_a/\tau}}{\sum_{b=1}^n e^{u_b/\tau}}$
- 18: **end for**

214 Evaluation of each strategy is complicated by the multiobjective nature
 215 of the rewards, which has two implications for experiments. First, it is nec-
 216 essary to evaluate each method’s performance across a range of settings for
 217 the action-selection operation (in our case, the thresholds used by the TLO
 218 operation). This ensures that no bias is introduced by a method favouring
 219 a particular region of objective space. Secondly, comparison of methods is
 220 easier if the average vector rewards received during the offline and online
 221 phase can be represented as scalar values.

222 The approach previously proposed in the literature is to carry out several
 223 runs with different action-selection parameters, and then to summarise the
 224 set of vector results achieved via the hypervolume metric [39]. This approach
 225 is well suited if the actual Pareto front of policies for the test environment is
 226 not known. However in the case of the benchmark environments used in this
 227 study, the true values of the Pareto-optimal policies are known. Therefore
 228 we can more explicitly measure the ability of the agent to satisfy the desired
 229 criteria by comparing the results achieved by the agent against the optimal
 230 policy for the action-selection settings, as advocated by [47]. As illustrated in
 231 Figure 2, for each Pareto-optimal policy of a particular environment (P_A , P_B
 232 and P_C), a corresponding set of TLO threshold parameters can be determined
 233 which would result in the selection of that policy (T_A , T_B and T_C). For each of
 234 the threshold settings, the learning algorithm is executed and its performance
 235 on each objective is measured (a_A , a_B and a_C). The agent’s mean reward
 236 is then compared to the expected per-episode reward for the corresponding
 237 Pareto-optimal policy. The vector difference between the expected and actual
 238 reward is then summarised as a scalar value using the additive-epsilon metric
 239 ($+\epsilon$) as proposed by [47]. This process is carried out for both the agent’s
 240 online and offline performance.

241 Note that as shown by points a_A and a_B in Figure 2, the agent’s perfor-
 242 mance may lie either above or below the thresholds – however this does not
 243 affect the calculation of the additive-epsilon metric. It is also possible, as
 244 shown by point a_C , that the agent may in fact outperform the optimal policy
 245 on one of the objectives. This positive difference between the agent and the
 246 optimal policy is simply ignored when calculating the additive epsilon metric.

247 For each of the exploration methods, this process is repeated over 20
 248 independent trials and further summarised as an online mean and offline
 249 mean over those trials. This is repeated for multiple parameter settings for
 250 each method, to investigate the sensitivity of each method to its parameters.
 251 For each benchmark a total of 5000 online (learning) episodes were executed

Figure 2: An illustration of the error measuring process used in the experiments. Dotted vertical lines indicate possible thresholds applied to the first objective, dark points indicate the corresponding Pareto-optimal policy for each threshold, and light points indicate the objective-space performance of an agent. The overall performance of this agent is the mean of the additive-epsilon error metrics E_A , E_B and E_C .

in each trial, with one additional episode executed in strictly greedy fashion to evaluate the offline performance. The learning rate α was set to 0.9 as was the eligibility trace parameter λ – these values were based on those utilised in prior research on the DST benchmark [19] and no attempt was made to tune these parameters for any of the individual methods. The exploration parameters were linearly decayed from their initial to final value over the online episodes – for ϵ the final value was 0, while for τ the final value was 0.01¹.

The experimental methodology is summarised below:

```

for each objective benchmark do
  for 20 independent trials do
    for each Pareto-optimal policy of that benchmark do
      set the TLO parameters to the thresholds matching that policy
      for 5000 learning episodes do
        observe the reward vector received over the episode
        calculate the  $+\epsilon$  difference w.r.t the Pareto-optimal reward

```

¹While not widely noted in the literature, numerical overflow errors can occur as the temperature τ nears zero. Limiting τ to a minimum value of 0.01 was sufficient to produce greedy behaviour by the end of training, while avoiding overflow errors.

```

268         end for
269         calculate  $+\epsilon$  for the final greedy policy (the offline performance)
270     end for
271 end for
272 report the mean online and offline  $+\epsilon$  over all trials and thresholds
273 end for

```

274 In order for an empirical study to be of value the methods being inves-
275 tigated must be evaluated across a set of test environments, exhibiting the
276 range of characteristics which might be encountered in practical application
277 of the methods. This is difficult for MORL research due to the paucity of
278 established benchmarks [39]. In addition, multiobjective Q-steering using
279 TLO action-selection has been shown to suffer convergence problems when
280 presented with environments in which more than one objective has non-zero
281 rewards at non-terminal states [19]. Such problems are common to all TD
282 algorithms using non-linear forms of action selection, due to the non-additive
283 nature of the returns in this situation [5].

284 As the aim of this research is to evaluate the performance of the explo-
285 ration methods rather than that of TLO or other non-linear approaches to
286 action selection, it would be potentially misleading to include benchmarks
287 with characteristics which inhibit TLO-based learning. Of the existing bench-
288 marks in the literature, the widely-used Deep Sea Treasure problem is suit-
289 able for use in conjunction with TLO [39]. For this study we have introduced
290 two further benchmark environments with characteristics which are suited to
291 TLO. The details of each benchmark are given below.

292 4.1. Deep Sea Treasure

293 [39] proposes several benchmarks for empirical MORL, including the Deep
294 Sea Treasure (DST) task (Figure 3). The DST has been widely adopted
295 as a benchmark (e.g. [15, 32, 43, 44]). The agent controls a submarine
296 which starts from a location near the shore and travels out to sea to retrieve
297 treasure. A reward of -1 is awarded for the time objective on each time-step,
298 while the treasure reward is zero except when a treasure is encountered at
299 which point it is equal to the value of that treasure (as shown in the grey
300 cells in Figure 3). Therefore the agent must find a trade-off between the
301 value of the treasure and the time taken to reach it. Under TLO this can
302 be achieved by minimising the time penalty, subject to achieving a threshold
303 value of treasure. The thresholds required to find each of the ten Pareto-
304 optimal policies are detailed in Table 1. This is an episodic task with no

305 discounting. Episodes end when a treasure is recovered or after a maximum
 306 of 1000 time-steps.

Figure 3: The Deep Sea Treasure task. Treasure values are indicated in the grey cells.

Threshold (for treasure objective)	Optimal policy per-episode reward (treasure, time)
0	(1, -1)
1.5	(2, -3)
2.5	(3, -5)
4	(5, -7)
6	(8, -8)
10	(16, -9)
20	(24, -13)
40	(50, -14)
60	(74, -17)
100	(124, -19)

Table 1: The set of Pareto-optimal policies and the corresponding TLO threshold parameter settings for the DST environment

307 4.2. Bonus World

308 Bonus World (Figure 4) is a new MORL benchmark introduced in this
 309 paper. Like DST it is a 2D episodic grid environment, but it has three
 310 objectives rather than two. Each episode starts with the agent in the location
 311 marked 'S'. The agent can move in the four cardinal directions, and receives
 312 a reward of -1 for the time objective on every time-step. When reaching a
 313 terminal state (shown by grey cells) the agent receives the rewards specified
 314 in that cell for the other two objectives. In addition the rewards in the

terminal states are doubled in magnitude if the agent has activated the bonus by visiting the cell marked 'X2'. The black cells near the bonus indicate walls which the agent can not pass through. Similarly the agent can not leave the bounds of the grid. Finally the cells marked 'PIT' indicate pits – if the agent enters one of these cells the bonus is deactivated, and the agent returns to the start state. A tabular representation of this environment has 162 discrete states – 81 for the cells of the grid when the agent has not activated the bonus, and 81 for the same cells when the bonus has been activated. The set of Pareto-optimal policies and the corresponding thresholds are listed in Table 2. It can be seen that trade-offs exist between all three objectives. Note that not all optimal policies require the agent to activate the bonus.

S								(1,9)
							PIT	
								(3,9)
			X2				PIT	
								(5,9)
							PIT	
								(7,9)
	PIT		PIT		PIT			
(9,1)		(9,3)		(9,5)		(9,7)		(9,9)

Figure 4: The Bonus World task. Terminal reward values for objectives 1 and 2 are indicated in the grey cells, while a reward of -1 is received for the third objective (time) on each step. S marks the start location, X2 marks the location of the bonus, black cells indicate walls, and PIT marks the presence of a pit which returns the agent to the start location.

4.3. Space Exploration

The third benchmark is the Space Exploration task (Figure 5). The agent controls a spaceship which starts each episode in the location marked 'S' and aims to discover a habitable planet while minimising the amount of radiation to which it is exposed. A penalty of -1 is received for the radiation objective on all time-steps, except when in a region of high radiation (marked 'R') when the penalty is -11. A positive reward is received for the mission success objective whenever a terminal state corresponding to a planet is reached – the magnitude of this reward reflects the desirability of that planet. If the ship enters a cell occupied by an asteroid, the ship is destroyed, the episode ends, and the agent receives a mission success reward of -100. The threshold is applied to the mission success objective, meaning that the agent will attempt to minimise radiation exposure subject to meeting minimum habitability requirements.

Thresholds (objective 1, objective 2)	Optimal policy per-episode reward (objective 1, objective 2, time)
(0, 8)	(1, 9, -8)
(2, 8)	(3, 9, -10)
(4, 8)	(5, 9, -12)
(10, 12)	(10, 18, -14)
(12, 16)	(14, 18, -16)
(16, 16)	(18, 18, -18)
(16, 12)	(18, 14, -16)
(12, 10)	(18, 10, -14)
(8, 4)	(9, 5, -12)
(8, 2)	(9, 3, -10)
(8, 0)	(9, 1, -8)

Table 2: The set of Pareto-optimal policies and the corresponding TLO threshold parameter settings for the Bonus World environment

340 Unlike the other two environments, in Space Exploration the agent can
341 move to all eight neighbouring states (i.e. there are eight actions). Also if
342 the agent leaves the bounds of the grid, it moves to the opposite edge of the
343 grid. For example if the agent moves up from the top row of the grid, it
344 will move to the bottom row of the same column). This means that, unlike
345 the other two benchmark environments, the execution of any action always
346 results in a change in environmental state. The Pareto-optimal policies for
347 this task and the corresponding thresholds are listed in Table 3.

20	R							10	R	R	30
20	R							10	R	R	30
20	R				S			10	R	R	30
20	R								R	R	30
20	R								R	R	30

Figure 5: The Space Exploration task. S marks the start location, while grey cells represent the terminal states corresponding to planets, and black cells are terminal states corresponding to asteroids. Cells marked R have higher than normal levels of radiation.

348 4.4. Comparison of Benchmarks

349 The primary characteristic which differs between the three benchmark
350 environments is the extent to which incorrect action choices can lead to
351 states which are 'catastrophic' for the agent. The Space Exploration task

Threshold for mission success objective	Optimal policy per-episode reward (mission success, radiation)
5	(10, -4)
15	(20, -15)
25	(30, -27)

Table 3: The set of Pareto-optimal policies and the corresponding TLO threshold parameter settings for the Space Exploration environment

is the most demanding with regards to this factor as the asteroid states result in immediate termination of the episode, and a reward which is very poor with respect to any of the Pareto-optimal policies. These states are also numerous and positioned within a single step of the optimal paths within the environment. The next most difficult environment is the DST, as terminal states exist which are close to the optimal path and which are sub-optimal with respect to most threshold settings – in particular the terminal state immediately below the starting state. The Bonus World problem is less challenging as each terminal state is generally at least two steps away from the optimal path for the other terminal states, and the pit locations which are within a single-step of the optimal path do not lead to episode termination, merely a penalty with regards to the time objective.

5. Experimental Results

5.1. ϵ -greedy exploration results

ϵ -greedy exploration is both the simplest form of exploration, and also the most widely-used in the MORL literature. Therefore the results achieved using this approach will be presented first as a baseline for comparison with the other approaches. Figure 6 illustrates both the online and offline error achieved when applying ϵ -greedy exploration to each of the benchmarks with a range of different settings for the initial value of ϵ . It can be seen that for the DST and Bonus World tasks this approach to exploration failed to find the optimal offline results on a consistent basis unless ϵ was set to a very high initial value. On the Space Exploration task, the offline performance was even poorer with no parameter setting being sufficient to guarantee finding the correct optimal policy.

The high initial values of ϵ required to optimise offline performance give rise to poorer online performance, as evident by the widening gap between

the online and offline performance on the right-hand side of Figure 6. The initial setting of ϵ required to optimise online performance on each task is considerably lower than that required to optimise the offline performance. The issue here is that all problems contain terminal states which are not optimal with regards to the current TLO threshold settings. The random nature of the exploration produced by the ϵ -greedy strategy is likely to encounter these states quite frequently. Entering one of these states results in a sub-optimal reward for the current episode. It also ends that episode, thereby limiting the agent’s ability to learn from that episode.

5.2. Softmax exploration results

In contrast to the results observed for ϵ -greedy exploration, both of the softmax exploration techniques were successfully able to learn the optimal offline policy for all three environments, as shown in Figures 7 and 8. In general this was achieved at initial temperature settings corresponding to relatively low levels of exploration, therefore resulting in quite good online performance.

Direct comparison of the results in Figures 6 to 8 is difficult due to the differing parameters (and therefore different horizontal axes used in the figures). However as discussed earlier in the paper, and as is evident in these figures, the performance of each algorithm across much of its parameter-space can be seen as a trade-off between online and offline performance². For any algorithm, the results achieved for a particular parameter setting can be treated as a point in a two-dimensional space, where one axis corresponds to online performance and the other to offline performance. By collating these points across all parameter settings for that algorithm and discarding any points which are Pareto-dominated, an approximate Pareto front can be produced. In this way the nature of the exploration-exploitation trade-off achieved by each algorithm can be visualised, as shown in Figure 9. By abstracting away from the parameters shown in Figures 6 to 8, these Pareto front graphs facilitate comparison of the performance of the different algorithms³.

²That is, exploration in RL is itself a multiobjective task.

³It should be noted that these figures, are not strictly Pareto fronts – both here, and in similar figures later in this paper, a small number of weakly dominated points have been included for purposes of clarity.

410 It can be seen that both softmax methods clearly dominate ϵ -greedy ex-
 411 ploration on the DST and Space Exploration tasks. Both the DST and Space
 412 Exploration tasks exhibit 'failure' states located a single step away from the
 413 optimal path, which are more likely to be encountered via the random explo-
 414 ration of ϵ -greedy than by the more directed softmax methods. In contrast
 415 the non-optimal terminal states in the Bonus World task are at least two steps
 416 away from the optimal path. This may explain why ϵ -greedy and softmax-t
 417 have similar fronts on this task, although the latter is still slightly superior.

418 Softmax-E is clearly superior to softmax-t on the Bonus World and Space
 419 Exploration tasks, but slightly weaker on the Deep Sea Treasure environment.

420 5.3. The effect of optimistic initialisation

421 The results reported in Section 5.2 were obtained using the default ini-
 422 tialisation of 0 for all objectives prior to the commencement of learning. We
 423 will refer to these results as those achieved using *neutral initialisation*, to dis-
 424 tinguish them from the *optimistic initialisation* considered in this section⁴.
 425 The optimistic initial values used for all of the experiments reported in this
 426 section are listed in Table 4.

Environment	Initial values for each objective
DST	(150, 0)
Bonus World	(20, 20, 0)
Space Exploration	(40, 0)

Table 4: The initial value vectors used for each benchmark environment for all experiments involving optimistic initialisation.

427 In the single-objective RL literature, optimistic initialisation is often used
 428 in conjunction with greedy action selection [23]. That is, the optimistic na-
 429 ture of the initial values is the sole factor driving exploration. However the
 430 results shown in Table 5 indicate that this approach was extremely unsucces-
 431 ful when applied in the context of Q-learning with multiple objectives and
 432 TLO action-selection, with both the online and offline performance being
 433 very poor.

⁴Note that 0 is not necessarily neutral for all objectives. For example on DST it is optimistic with respect to the time objective, and pessimistic with respect to the treasure objective. Truly neutral initialisation can only be achieved if the exact values of the optimal policy are known in advance, which will generally not be the case.

Environment	Online mean	Offline mean
DST	578.2	579.0
Bonus World	306.4	306.5
Space Exploration	979.0	983.0

Table 5: The mean online and offline results achieved using greedy exploration with optimistic initialisation on all three benchmarks.

434 A closer examination of the behaviour of the agent during these trials
 435 revealed that the cause of this performance was the agent becoming caught
 436 in loops of recurring states and actions. That is to say, once the agent reached
 437 a particular state s it would execute a sequence of one or more actions which
 438 lead back to the same state s . The combination of optimistic initialisation,
 439 the reward structure of the environments, and the non-linear nature of the
 440 TLO action-selection could potentially result in that same sequence of actions
 441 continuing to be regarded as greedy by the agent regardless of how many
 442 times the sequence has been executed.

443 This is best illustrated by an example. Consider the case where the agent
 444 is currently occupying a state in the upper row of the DST, and the threshold
 445 for the treasure objective has been set at 100. Assume that all actions other
 446 than moving up have previously been explored and eventually lead to terminal
 447 states, so their Q-values have been trained downwards (e.g. to (50,-6))
 448 while the Q-values for the up action remain at their initial setting of (150,0).
 449 Clearly up will be regarded as the greedy action, and if executed will result
 450 in a reward of (0, -1) and the agent remaining in the same state. As the
 451 agent remains in the same state, no discounting is being applied, and the
 452 immediate reward for the treasure objective is zero, the estimated Q-value
 453 for this state-action pair for the treasure objective will remain unchanged. In
 454 contrast the immediate reward of -1 for the time objective will result in the
 455 Q-value for that objective being lowered. Assuming α of 0.9, the updated
 456 Q-value for this action will now be (150, -0.9). As the estimated treasure
 457 value remains above the threshold, this action will continue to be regarded
 458 as the greedy action. No matter how many times the action is executed it
 459 will remain the greedy choice, as the Q-value for the treasure objective is
 460 never trained downwards. The estimated value of the time objective for this
 461 action will steadily train downwards – however in this situation TLO does
 462 not take this objective into account as this is the only action exceeding the
 463 threshold value for the first objective. Therefore this action will be repeatedly

464 executed until the agent chooses an exploratory action. This leads to poor
 465 online performance, and also can result in the agent failing to find the opti-
 466 mal offline policy. While the example given here consists of a single action,
 467 recurrent sequences of two or more actions were also observed. In particular
 468 the structure of the Space Exploration task is such that a single action can
 469 not leave the environment’s state unchanged, but recurrent behaviour was
 470 still evident in the results on this task.

471 The results in Table 5 clearly show that the combination of optimistic ini-
 472 tialisation and greedy action-selection is not adequate for the tasks examined
 473 in this paper. However, optimistic initialisation may still be beneficial when
 474 combined with some other form of exploration. Figures 10 to 12 compare
 475 the performance of each exploration method under neutral and optimistic
 476 initialisation. It can be seen that the incorporation of even a small amount
 477 of additional exploration in conjunction with optimistic initialisation is suf-
 478 ficient to overcome the problem highlighted in Table 5, and allow learning of
 479 the optimal offline policy.

480 When the performance of each algorithm with neutral initialisation is
 481 compared against the same algorithm using optimistic initialisation (Figures
 482 10 to 12), it can be seen that the optimistically initialised agent is generally
 483 able to learn the optimal offline policy at much lower settings for the explo-
 484 ration parameter (ϵ or τ), which in turn leads to marked improvements in
 485 the online performance. The only exception is for the combination of soft-
 486 max exploration with very low initial values of τ where the optimistically
 487 initialised agent still exhibits signs of the problematic behaviour observed for
 488 greedy action-selection. Softmax-E performed more poorly than softmax-t
 489 under these conditions as reflected by the different vertical scales required
 490 for the DST and Space Exploration graphs in Figures 11 and 12. The reason
 491 is that under optimistic initialisation very large differences in Q-values can
 492 easily arise early in learning. For example in the Space Exploration task,
 493 if most actions from the current state have been found to lead to asteroids,
 494 then any untried actions will be valued much more highly than those actions
 495 which have been experienced. By focusing only on the rankings of actions
 496 rather than the magnitude of the differences, softmax-t somewhat reduces
 497 the impact of these early variations in values when compared to softmax-E.

Figure 6: The mean online and offline error achieved using ϵ -greedy exploration on the Deep Sea Treasure (top), Bonus World (middle), and Space Exploration (bottom) environments.

Figure 7: The mean online and offline error achieved using softmax-t exploration on the Deep Sea Treasure (top), Bonus World (middle), and Space Exploration (bottom) environments.

Figure 8: The mean online and offline error achieved using softmax-E exploration on the Deep Sea Treasure (top), Bonus World (middle), and Space Exploration (bottom) environments.

Figure 9: Pareto-front visualisation of the tradeoff between mean online and offline error achieved using ϵ -greedy exploration and both variants of softmax exploration on the Deep Sea Treasure (top), Bonus World (middle), and Space Exploration (bottom) environments.

Figure 10: The mean online and offline error achieved using ϵ -greedy exploration with both neutral and optimistic initialisation on the Deep Sea Treasure (top), Bonus World (middle), and Space Exploration (bottom) environments.

Figure 11: The mean online and offline error achieved using softmax-t exploration with both neutral and optimistic initialisation on the Deep Sea Treasure (top), Bonus World (middle), and Space Exploration (bottom) environments.

Figure 12: The mean online and offline error achieved using softmax-E exploration with both neutral and optimistic initialisation on the Deep Sea Treasure (top), Bonus World (middle), and Space Exploration (bottom) environments.

498 The impact of optimistic initialisation is further illustrated by the Pareto
 499 front visualisations in Figures 13 to 15. It can be seen that for every ex-
 500 ploration strategy and every environment, the Pareto front resulting from
 501 optimistic initialisation clearly dominates that produced using neutral ini-
 502 tialisation.

Figure 13: Pareto-front visualisation of the tradeoff between mean online and offline error achieved using ϵ -greedy exploration with both neutral and optimistic initialisation on the Deep Sea Treasure (top), Bonus World (middle), and Space Exploration (bottom) environments.

Figure 14: Pareto-front visualisation of the tradeoff between mean online and offline error achieved using softmax-t exploration with both neutral and optimistic initialisation on the Deep Sea Treasure (top), Bonus World (middle), and Space Exploration (bottom) environments.

Figure 15: Pareto-front visualisation of the tradeoff between mean online and offline error achieved using softmax-E exploration with both neutral and optimistic initialisation on the Deep Sea Treasure (top), Bonus World (middle), and Space Exploration (bottom) environments.

Figure 16: Pareto-front visualisation of the tradeoff between mean online and offline error achieved using ϵ -greedy exploration and both variants of softmax exploration with optimistic initialisation on the Deep Sea Treasure (top), Bonus World (middle), and Space Exploration (bottom) environments.

Figure 16 provides a comparison of the overall performance of ϵ -greedy, softmax-t and softmax-E exploration across the three benchmark problems, using optimistic initialisation. On both the DST and Space Exploration environments both softmax methods clearly outperform ϵ -greedy exploration. Even though the performance of ϵ -greedy on these problems is improved by optimistic initialisation, the random exploration still encounters the 'failure' states more frequently than for the softmax methods. The results on the Bonus World task are less clear-cut, with the ϵ -greedy and softmax-E fronts having similar end-points – this environment lacks the catastrophic failure

512 states which impede ϵ -greedy’s performance on the other environments. Over
513 all three environments, softmax-E appears to perform slightly better than
514 softmax-t.

515 6. Conclusion and Future Work

516 This work has established several benchmarks for evaluating exploration
517 in MORL, and has used these to highlight issues which may arise when ap-
518 plying the simple exploration methods commonly used in single-objective RL
519 in a multiobjective context. In particular it has been shown that the simple
520 combination of optimistic initialisation and greedy action-selection which has
521 been widely used in single-objective RL performs extremely poorly when ap-
522 plied to multiobjective problems in combination with the TLO approach to
523 action-selection. The combination of the environmental reward structure and
524 TLO can lead to the agent repeatedly executing recurrent series of action(s)
525 and therefore failing to learn the optimal policy. However the combination of
526 optimistic initialisation with other forms of exploration can be very effective.

527 We have shown that the softmax approach to exploration can be extended
528 to handle multiple objectives. Of the two variants examined in this work, the
529 softmax-E method (which combines softmax exploration with the additive-
530 epsilon metric) appears to provide the most effective trade-off between explo-
531 ration and exploitation. In particular this approach in combination with opti-
532 mistic initialisation produced strong online and offline performance across all
533 three benchmark environments. While the results reported here are specific
534 to TLO action-selection, the softmax-E approach would be equally applicable
535 in combination with any non-linear form of action-selection.

536 Future work needs to examine whether more sophisticated approaches
537 to exploration can be adapted or developed for multiobjective RL. One
538 promising direction is the Exploration Objective Value Function (EOVF) ap-
539 proach recently proposed by [48]. This approach essentially ‘multiobjectifies’
540 a single-objective problem by adding an additional, optimistically initialised
541 ‘novelty’ objective. This receives a reward of zero on all time-steps and is
542 discounted so that its values eventually decay to zero. The separation of op-
543 timistic initialisation into a separate objective may address some of the issues
544 observed in this paper. However [48] use a linear weighted combination of
545 objectives, so it remains to be seen how EOVF will function in the context
546 of non-linear action selection.

547 An additional line of further investigation will be to examine how well
 548 various exploration methods operate for multi-policy MORL algorithms such
 549 as the Pareto set approach of [37]. In order to find a wide range of Pareto-
 550 optimal policies, multi-policy methods are likely to require different forms or
 551 levels of exploration than single-policy approaches, but this issue has yet to
 552 be properly investigated.

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