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Multi-modal reliability analysis of slope stability

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Abstract

Probabilistic slope stability analysis typically requires an optimisation technique to locate the most probable slip surface. However, for many slopes particularly those containing many different soil layers or benches several distinct critical slip surfaces may exist. Furthermore, in large slopes these critical slip surfaces may be located at significant distances from each other. In such circumstances, finding and rehabilitating the most probable failure surface is of little merit, as rehabilitating that surface does not improve the safety of the slope as a whole. Unfortunately, existing slip surface search techniques were developed to converge on one global minimum. Therefore, to implement such methods to evaluate the stability of a slope with multiple failure mechanisms requires the user to define probable slip locations prior to calculation. This requires extensive engineering experience and places undue responsibility on the engineer in question. This paper proposes the use of a locally informed particle swarm optimisation method which is able to simultaneously converge to multiple critical slip surfaces. This optimisation model when combined with a reliability analysis is able to define all areas of concern within a slope. A case study of a railway slope is presented which highlights the benefits of the model over single objective optimisation models. The approach is of particular benefit when evaluating the stability of large existing slopes with complicated stratigraphy as these slopes are likely to contain multiple viable slip surfaces.

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1. Introduction

Nomenclature

FOS	Factor of Safety
c'	Effective cohesion
Δx	slice width
W	slice weight
ϕ'	Friction angle of soil
u	Porewater pressure
α	Inclination of slice base
β	Reliability Index
g(X)	Performance function
E(X)	Mean of variable X
$\sigma(X)$	Standard deviation of X
X	Vector of variables
P_f	Probability of failure
$P_{f,sys}$	System Probability of failure
ρ	Correlation matrix
U	Particle position
V	particle velocity

Slope instability is one of the main problems faced by transport networks. Traditionally earthworks were designed to resist deep seated rotational failures. However, over recent years increased rainfall across much of Western Europe, has led to a sharp increase in incidence of shallow planar failures. These failures are caused by infiltrating rainwater percolating downward filling available soil pore space thereby reducing inherent soil suctions and temporarily lowering the shear strength of the near surface soils (Ridley et al. 2004; Xue & Gavin 2007).

This is of particular concern for aged transport networks such as the rail networks across both Ireland and the UK where many earthwork assets were constructed in the mid-19th century. These slopes are typically far steeper than those recommended by modern design standards and as a result many rely on soil suctions for stability (Jennings & Muldoon 2001). This makes them more susceptible to changes in climatic condition. Given the current economic circumstances throughout Europe, the budget is not available to replace all offending slopes. However it is imperative that we are able to identify and remediate the slopes which represent the biggest risk to end-users, as any failure could potentially result in numerous fatalities.

Probabilistic methods are ideally suited for evaluating existing infrastructure as you can account for parameter variation, both spatially and temporally, by modelling each variable using a distribution or in some cases a series of distributions. This allows for a much more realistic estimate of a slopes capacity than traditional deterministic design, where the designer selects single parameter values to represent strength properties throughout the slope. Typically this results in ultra-conservative designs as the designer is forced into selecting lower bound values in order to reduce risk, however in some extremely variable soils even this approach may not be adequately conservative enough. Furthermore, using a deterministic methodology many safe slopes get misclassified as areas of concern. Therefore this paper uses reliability theory to evaluate the stability of slopes, outputting a probability of failure as opposed to the factor of safety more traditionally seen in slope design.

Furthermore given that slopes are susceptible to many different forms of failure as outlined above this paper evaluates stability using a multi-modal optimization algorithm (LIPS) which is able to detect all viable slip surfaces simultaneously. This algorithm is demonstrated in conjunction with Bishops circular method and first order system reliability method to assess the stability of transport slopes.

2. Methodology

2.1. Traditional Limit State Equation

Traditionally slope stability was evaluated in terms of a factor of safety, which was obtained by inputting deterministic parameter values into limit equilibrium equations in the form of Equation (1), where the factor of safety of a slope can be defined as the ratio between resistance and disturbance along a potential slip surface:

$$FOS = \frac{resistance}{disturbance} \tag{1}$$

There are many such methods published (Fredlund & Krahn 1977) each with their own positives and negatives most of which are based on the method of slices, where the slope is divided into a number of vertical slices and the forces/moments for the slope are determined about the origin. . For the slope shown in Figure 1, using the simplified Bishop's method of slices (Bishop 1955), the factor of safety of a slope can be defined as:

$$FOS = \frac{\sum_{i=1}^{n}[c_i \Delta x_i + (W_i - u_i \Delta x_i)\tan\phi_i]\frac{sec\alpha_i}{1+\tan\phi_i \tan\alpha_i/FOS}}{\sum_{i=1}^{n} W_i \sin\alpha_i} \tag{2}$$

where W_i is the weight of the i^{th} slice, α_i is the tangential angle of the base of the i^{th} slice, Δx_i is the i^{th} slice width, c_i is the cohesion of the soil on the base of the i^{th} slice, u_i is the pore water pressure at the base of the i^{th} slice, and ϕ_i is the friction angle of the soil at the base of the i^{th} slice. To obtain the minimum FOS of a slope, either a trial and error or an optimization technique must be implemented.

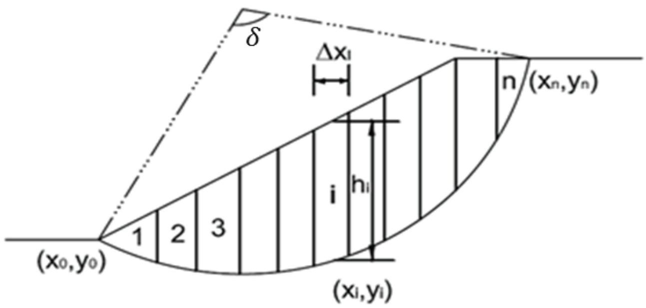

Fig. 1. Terms used to describe slip surface geometry.

2.2. Probabilistic methods

Over recent decades probabilistic methods have become increasingly common across Transport Engineering. One area in particular which has received significant attention is slope stability(Xu & Low 2006; Gavin & Xue 2010; Cheng et al. 2015; Liang et al. 1999; Reale et al. 2016). This is due to researchers recognizing the inadequacy of deterministic design in light of the significant uncertainties associated with site investigation, slip surface location, climate and of course spatial variation. Reliability analyses assign distributions to each variable allowing any uncertainties to be accounted for within stability calculations, thereby offering a more meaningful rational interpretation of slope safety over traditional deterministic design which assumes fixed point values. The performance function g(X) or limit state function of a slope can be expressed as the difference between a slopes capacity (C) and demand (D), see Equation 3.

$$g(X) = (C - D)\begin{cases} > 0, safe\ state \\ = 0, limit\ state \\ < 0, failure\ state \end{cases}$$

$$g(X) = g(x_1, x_2, \dots, x_n) for\ i = 1\ to\ n \tag{3}$$

where X is a vector of the different random variables (x_i) represented in the slope. Safety in a reliability analysis is typically expressed in terms of a reliability index, β, and a probability of failure, p_f. The probability of failure (p_f) can be defined as the probability at which the performance function is less than zero, see Equation 4.

$$P_f = P[g(X) \leq 0] \tag{4}$$

In a normal space, the reliability index (β) is defined as the distance in standard deviations from the mean of the performance function to the design point, Equation 5. This can be seen graphically in Figure 2.

$$\beta = \frac{E[g(X)]}{\sigma[g(X)]} \tag{5}$$

Where *E[g(X)]* is the mean of the performance function and *σ[g(X)]* is its standard deviation. When analysing slope stability the performance function of the slope is typically expressed as in Equation (6).

$$g(X) = FOS - 1.0 \tag{6}$$

where FOS is the factor of safety as defined by a relevant limit state equation.

There are at present a number of means of performing a probabilistic analysis of Equation (6). These can be separated into two distinct groups approximate methods such as FOSM (first order second moment) etc. and simulation methods such as Monte Carlo.

This paper utilizes an approximate method namely first order reliability method more commonly known as FORM. This approach is discussed below.

2.3. First Order Reliability Methods Hasofer Lind

Hasofer & Lind (1974) proposed an approximate method which assumes a first order tangent to the limit state function at the design point (i.e. when g(X)=0). This method gives an exact solution for linear performance functions and a close approximation for nonlinear functions. This method known as FORM is commonly used across many engineering disciplines. FORM requires all computation to be done in the standard normal space, therefore the vector of uncorrelated random variables (X) needs to be transformed into a vector of standardised normal variables ($\bar{X}$) prior to minimisation. Equation (7) is used to transform random variables into the standard normal space.

$$\bar{X}_i = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}}\ \ for\ \ i = [1,2, \dots, n] \tag{7}$$

The reliability index is then defined as the minimum distance from the origin to the limit state surface in the normalised Gaussian space, See Equation 8.

$$\beta = min_{\bar{X} \epsilon \psi}\{\bar{X}\bar{X}^T\}^{1/2} \tag{8}$$

where the limit state surface *Ψ* is defined by $g(\bar{X}) = 0$.

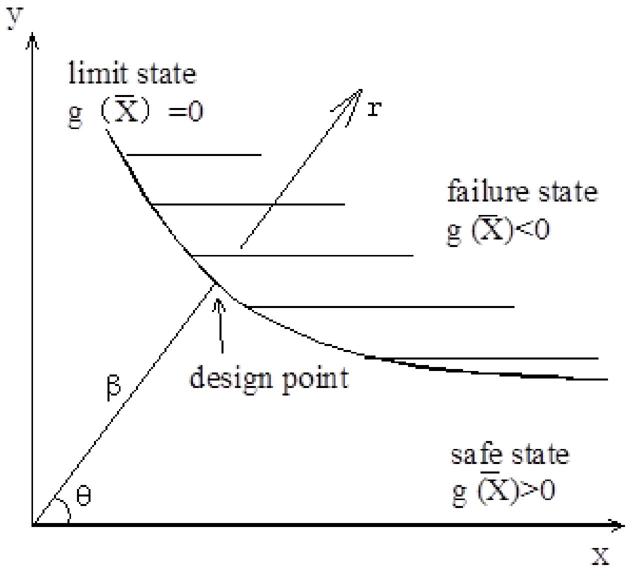

Fig. 2. Hasofer Lind reliability index shown graphically as the minimum distance from the origin to the limit state surface in a reduced normal space.

2.4. System Reliability

As slopes are susceptible to many different failure mechanisms the actual failure surface is usually dependant on whichever triggering mechanism which presents itself first. i.e. a severe weather event is likely to trigger a shallow slide while additional physical loading will preferentially deteriorate a deep seated failure. As a result, slope stability is increasingly being considered as a system reliability problem, where the probability of any aspect of the slope failing is considered as opposed to the probability of the critical slip surface failing. As the system failure probability incorporates the probability of the critical slip surface failing, the probability of the system failing is larger than that of any individual component. Therefore in order to evaluate system reliability all viable slip surfaces need to be identified and the correlation between them established. In a Cartesian co-ordinate defined search space the correlation between the different failure modes (ρ) can be determined by Equation (9), where $\bar{X}$ (see Equation 7) is a vector containing the design points of all possible failure modes.

$$\rho = \frac{\bar{X}^T \bar{X}}{\beta_i \beta_j} \tag{9}$$

While a number of different methodologies exist for evaluating the system reliability this paper utilises a bi-model bounded approach developed by Ditlevsen (1979) which estimates the system probability of failure based on the correlation between the different failure modes.

$$\begin{aligned} P_{f,max} + max \sum_{i=2}^{m} \left[P_f(i) - \sum_{j=1}^{i-1} P_f(i,j);\ 0 \right] &\leq P_{f,sys} \\ &\leq \min \left[\sum_{i=1}^{m} P_f(i) - \sum_{i=2}^{m} \max_{j<i} P_f(i,j);\ 1 \right] \end{aligned} \tag{10}$$

2.5. Optimisation method – Locally Informed Particle Swarm Optimisation (LIPS)

For multimodal problems, numerous extrema exist which need to be located simultaneously, these optima may be located in vastly different areas of the search space. This paper uses a multi modal optimisation algorithm termed LIPS (locally informed particle swarm) to locate all significant minima. LIPS is a modified form of particle swarm optimisation (PSO) adapted to solve multi-modal problems. PSO is based on how swarm animals such as birds seek food collaboratively. The general idea being that each individual animal within the swarm is considered as a particle within the algorithm and each particle represents a solution (collection of design points) to an optimisation problem. These particles then move about the search space or performance function surface with a certain velocity. During every iteration each particle updates both its velocity and its position based on both that particles best experience (lowest β or FOS) so far (termed *pbest*) and the swarms best experience so far (global optima termed *gbest*). When a particle nears an optima its velocity decreases. Each particle is aware of the current global best solution and if the program runs for long enough all particles should move towards this point.

LIPS differs from standard PSO in that not every particle is aware of the location of the global minimum at any given time, instead each particle is aware of its personal best solution and that of its neighbourhood. Where a particles neighbourhood, is the m closest particles to that particle measured in Euclidean distance. This allows particles to learn from those particles surrounding, while also mitigating the influence of particles on the opposite side of the search space. This ensures that LIPS is able to develop a number of stable niches in different areas of the search space thus allowing the algorithm to optimise simultaneously to multiple different local optima.

The velocities (V) and positions (U) of the particles are updated using Equations (14 - 16). Further details on the optimisation process can be found in Reale et al. (Reale et al. 2015).

$$U_{i,d}^{t+1} = U_{i,d}^{t} + V_{i,d}^{t} \tag{11}$$

$$V_{i,d}^{t+1} = \vartheta.\left(V_{i,d}^{t} + \varphi\left(P_{i,d}^{t} - U_{i,d}^{t}\right)\right) \tag{12}$$

$$P_{i,d}^{t} = \frac{\sum_{j=1}^{nsize}(\varphi_j \; .nbest_j)/nsize}{\varphi} \tag{13}$$

Where φ_j is a random distributed number in the range of [0, (4.1)/*nsize*] and φ is equal to the summation of φ_j . $nbest_j$ is the j^{th} nearest neighbourhood to i^{th} particle's personal best (*pbest)*, *nsize* is the neighbourhood size and ϑ is the inertia weight which balances the search between global and local performance.

3. Case Study – Rail Embankment

A 10 m rail embankment is used to demonstrate the capabilities of the method. The slope is comprised of a glacial till with a slope angle of 38 $^{\circ}$, while ground level is inclined at 2 $^{\circ}$ to the horizontal, see Figure 3. The bearing stratums underlying the embankment consist of a shallow weak silty clay layer overlying a stiffer clay deposit. The geotechnical parameters used are presented in Table 1.

Several failure modes were detected by the LIPS algorithm, see Figure 4, with two distinct failure mechanisms found, one shallow seated slip surface contained entirely within the embankment layer and another deeper failure surface passing through the underlying clay layer. The correlation matrix between the failure modes and their respective reliability indices is shown in Table 2. The probability of failure of the critical slip surface (m3) is 0.0016, while the system probability bounds were found to be [0.0017, 0.0027]. This demonstrates the importance of analyzing the slope as a system instead of just being concerned with the critical slip surface which may be significantly safer than the slope assessed as a whole.

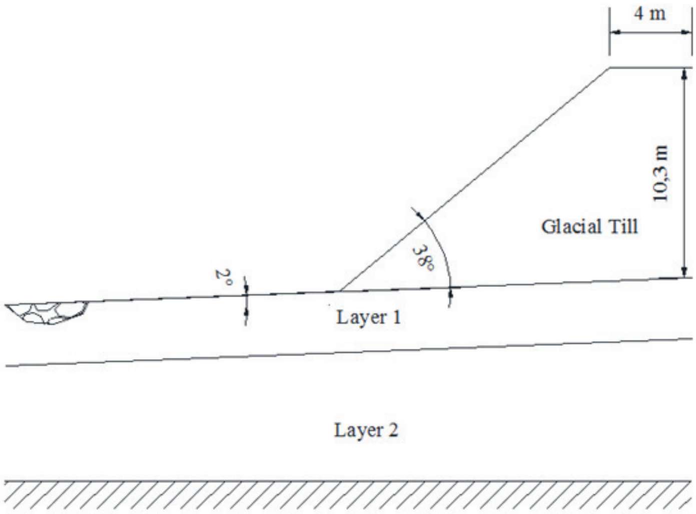

Fig. 3. Slope profile.

Table 1. Geotechnical parameters used in analysis.

Property	Mean	Coefficient of Variation
Cohesion (embankment) (kPa)	5	0.2
Friction angle (embankment) (°)	34	0.1
Cohesion (layer 1) (kPa)	10	0.2
Friction angle (layer 1) (°)	28	0.15
Cohesion (layer 2) (kPa)	20	0.1
Friction angle (layer 2) (°)	26	0.1

Table 2. Correlation matrix between different slip surfaces and associated reliability indices.

	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	β
ρ_1	1.00	0.64	0.55	0.71	0.65	3.80
ρ_2	0.64	1.00	0.96	0.98	0.98	3.24
ρ_3	0.55	0.96	1.00	0.96	0.94	2.95
ρ_4	0.71	0.98	0.96	1.00	0.99	3.27
ρ_5	0.65	0.98	0.94	0.99	1.00	3.02

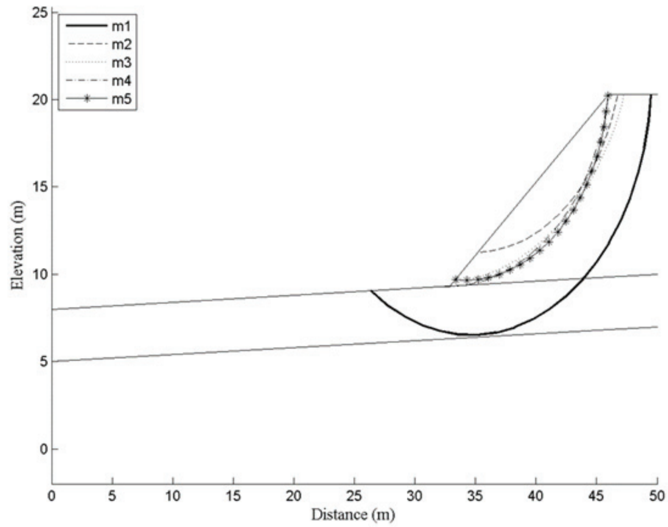

Fig. 4. Representative probabilistic slip surfaces detected by LIPS.

4. Conclusions and recommendations for future research

Across Europe railway Infrastructure managers are facing challenges in managing aged cutting and embankment assets with reduced budgets. Compounding this changes in climate are causing increased stress on these networks with slope capacity inherently linked to climate. This paper has shown the benefits of combining multi-modal optimisation algorithms with probabilistic methods for analysing existing railway slopes. A first order reliability method was used in conjunction with Bishops Circular method and a bi-modal bounded system reliability approach to determine the stability of the slope presented. The optimisation algorithm was able to detect the presence of all viable slip circles simultaneously thereby eliminating the chance of a missing a key slip surface, thus removing subjectivity caused by the designer. As can be seen from Figure 4 the LIPS model was able to detect both deep seated and shallow failures in the same slope. If a traditional analyses was performed only the shallow failure would have been detected. However depending on the proximity of the railway track to the embankments edge, the shallow failure may not be of critical importance to the Infrastructure manager, whilst the slightly safer larger rotational failure could be more concerning.

Probabilistic tools are extremely useful for evaluating aged infrastructure as they can account for uncertainties in building materials and both spatial and temporal variability, thereby allowing a far more accurate representation of slope capacity. Which in some cases can lead to assets previously classified as unstable by deterministic methods, being reclassified as safe due to low levels of uncertainty in the design. In such cases design lives can be extended allowing for substantial cost savings. This can be seen in the case study presented which has very steep sides (38 º) but has a minimum reliability index of approximately 3 which would be considered stable, however the associated minimum factor of safety is 1.15 which would be considered unacceptable.

Finally the authors wish to acknowledge that the model is currently limited to circular slip circles which may not be the critical slip surface for all situations i.e. in certain weather conditions parallel failures may take precedence.

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